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A STUDY OF THE EQUIVALENT CIRCUIT OF QUARTZ CRYSTALS

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C. C. Rust

A STUDY OF THE EQUIVALENT CIRCUIT OF QUARTZ CRYSTALS

by

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PREFACE

The period between 1 January, 1949, and 15 March, 1949, was spent by the author at the R.C.A. Crystal Research Laboratory at Camden, New Jersey. The Laboratory personnel consists of the following:

C. T. Beishline, Laboratory Technician

J. W. Conn, Wireman

L. L. Dimmick, Engineer

P. D. Gerber, Engineer

M. Shaffer, Laboratory Technician

S. Steltz, Toolmaker

E. M. Washburn, Engineer, Laboratory Manager

The division of personnel, and the equipment of the Laboratory, is such as to enable the group to completely fabricate a crystal unit of production or experimental design. This requires ability and equipment to grind to frequency, silver plate, bake at high temperatures, produce mounts and covers, design and assemble oscillator or other circuits; they can also perform highly accurate frequency measurements and impedance measurements in connection with tests involving operation over wide temperature variations, vibration effects, aging, and over-excitation effects.

The entire personnel of the R.C.A. Crystal Laboratory was entirely cooperative in familiarizing the author with their problems, equipment, and techniques. The manager, Mr. E. M. Washburn, encouraged work along many lines of inquiry of interest to the Armed Services; he made available to the author the facilities and engineering advice of his department in an investigation of general methods of frequency and impedance measurements of crystals at frequencies from 80KC to 60 MC, and the specific problem as to the effect of variations of the interpin capacity of crystals on their stability and activity.

Dr. W. D. George of the National Bureau of Standards has shown an inspiring understanding in the field of frequency and impedance measurement, and the author acknowledges the value of his inspiration and assistance.

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I. INTRODUCTION TO EQUIVALENT CIRCUIT AND MEASUREMENT OF ITS PARAMETERS.

Van Dyke (6) first showed in 1925 that a quartz crystal may be represented by an equivalent circuit such as shown in Fig. 1. His work was based on the fundamental differential equations given by Cady in 1922 (1) for the current to a piezo-electric resonator and for the forced vibrations of the resonator under an applied alternating potential. Van Dyke derived expressions for:

- L_1 in terms of equivalent mass, piezo-electric constant and dimensions of the crystal.
- C_1 in terms of piezo-electric constant, dimensions, and stiffness.
- R_1 in terms of frictional coefficient, piezo-electric constant, and dimensions.
- C_o in terms of dielectric constant, modulus, piezo-electric constant, and dimensions.

At this time application of crystals to what is now referred to as crystal oscillator circuits was being investigated. In 1918, A. M. Nicholson (5) filed patents covering use of crystals in conjunctions with vacuum tubes as oscillator circuits. Already it had been shown, however, that a quartz plate, from the laboratory of Cady, oscillating longitudinally at a fundamental frequency of 90KC, could be represented by an equivalent circuit with:

$L_1 = 140$ henries

$C_1 = .023$ uuf

$R_1 = 16{,}000$ ohms

$C_o = 3.6$ uuf.

With the widespread development in the field of crystals oscillators, it has become possible to measure experimentally the resonant and antiresonant frequencies of crystals, and calculate therefrom the value of L_1, C_1, R_1, and C_o of the equivalent circuit of the crystal. Of course, experimental values of the resonant and antiresonant frequencies did not agree exactly with the values predicted from the equivalent circuit derived in terms of mass, dimensions, elastic, piezo-electric, and other properties of the crystals. Because of this, a great deal of work has been done in the measurement of the physical constants of crystalline materials. An example of a leader in this endeavor is W. P. Mason (4) of the Bell Telephone Laboratories.

The interest of this paper does not lie in the measurements of elastic or piezo-electric constants or the deviation of the parameters of the equivalent circuit of the crystal therefrom. Rather, the purpose of this study is to accept the equivalent circuit of the crystal proposed by Cady and others, show some of the relations that exist among L_1, C_1, R_1 and C_o, show how the values of these parameters may be measured experimentally and used to predict the behavior of a crystal in its using circuit. Variations of the equivalent circuit parameters at the same and at different frequencies are shown. Finally, the equivalent circuit is considered, with emphasis on its general characteristics as a tank circuit.

Outstanding work along the lines enumerated at the end of the preceding paragraph has been done by C. W. Harrison (3) and others at Bell Telephone Laboratories and W. D. George (2) and others at the National Bureau of Standards.

However, crystals have always been and are still being procured on the basis of specifications written in terms of their performance in specified test circuits. Attempts are being made at present by Joint Army-Navy-Air Force Organizations (ANEESA) to write specifications, for crystal procurement based on measurement of parameters of the crystal itself, that will be acceptable to the Armed Services and industry. It seems a certainty that specifications will be so worded in the near future. This will have the effect of increasing the interest of the crystal engineers in thinking of the crystal in terms of its equivalent circuit parameters. They will seek more knowledge of how they may purposely vary L_1, C_1, R_1, or C_o. They will be called upon by designers to interpret what variations of these parameters mean in terms of performance -i.e., stability, activity, etc. They will want to know what variations may exist among "normal" crystals. Of course, these things are not completely unknown, but it is the purpose of this paper to furnish information which may be useful along the following lines of inquiry:

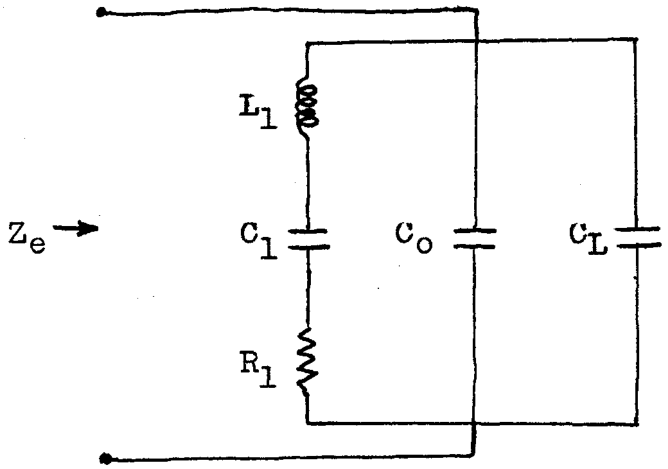
(1). How may the parameters of the equivalent circuits of crystals of low, medium, and high frequency(80KC to 60MC) be measured with fair accuracy with a reasonable amount of laboratory equipment?

(2). What is the effect of a variation of area of plating of plated crystals on the parameters of the equivalent circuit and what effect does this have on the stability and activity of the crystal?

(3). What variations may be expected in $L1, C_1, R1$ and C_o among commercial crystals of the same and of different frequencies?

II. EQUIVALENT CIRCUIT RELATIONS AND FORMULAE

This chapter is devoted to a derivation of a number of impedance and other relations involving the parameters of the equivalent circuit of the crystal. Nomenclature is as indicated on Fig. 1, or as explained in the text.



$C_t \quad C_o + C_L$

Fig. 1

A summary of the relations developed is given, followed by notes on their derivation:

(A) $C_o = \frac{.416\ A}{t_1}$ uuf interpin capacity for plated xtal.

A and t_1 in cm.

(B) $f_s = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1}}$ $\qquad f_a = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1} + \frac{1}{L_1 C_t}}$ $\qquad \Delta f = f_a - f_s$

(C$_1$) $$R_e = \frac{R_1}{\omega^2 C_t^2 \left[R_1^2 + \left(\omega L_1 - \frac{1}{\omega C_1} - \frac{1}{\omega C_t}\right)^2\right]}$$

$$(C_2)\quad X_e = \frac{\left(\omega L_1 - \frac{1}{\omega C_1}\right)\frac{1}{\omega^2 C_t^2} - \left[R_1^2 + \left(\omega L_1 - \frac{1}{\omega C_1}\right)^2\right]\frac{1}{\omega C_t}}{R_1^2 + \left(\omega L_1 - \frac{1}{\omega C_1} - \frac{1}{\omega C_t}\right)^2}$$

$$(D_1)\quad R_e \cong \frac{R_1}{\left(1 - \frac{2C_t}{C_1}\frac{\Delta f}{f_s}\right)^2}$$

$$(D_2)\quad X_e \cong \frac{4\pi L_1 \Delta f}{1 - \frac{2 C_t \Delta f}{C_1 f_s}}$$

$$(E2)\quad Z_e \text{ at } f_a \cong \frac{1}{\omega_a^2 C_t^2 R_1} = \text{Performance index.}$$

$$(E_2)\quad \Delta f = f_a - f_s = \frac{f_s C_1}{2(C_o + C_L)}$$

$$(F)\quad R_e \cong R_1 \left(1 + \frac{C_o}{C_L}\right)^2$$

$$(G)\quad L_1 \cong \frac{1}{8 \pi^2 f \Delta f C_t}$$

$$(G_2)\quad C_1 \cong \frac{2 \Delta f C_o}{f}$$

$$(H)\quad Q = \frac{\omega L_1}{R_1} \cong \frac{1}{4 \pi R_1 C_o \Delta f}$$

(I) $$\gamma = \frac{C_o}{C_1} = \frac{f}{2\,\Delta f}$$

(J) $$\frac{d\,f_a}{d\,C_t} = \frac{-\,f\,C_1}{2\,C_t^{\,2}} = \frac{-\,f\,C_1}{2(C_o + C_L)^2}$$

(K$_1$) $$C_t\ \text{max} \cong \frac{1}{4\pi f R_1}$$, if f_a is to occur.

(K$_2$) $$R_1\ \text{max} \cong \frac{1}{4\pi f C_t}$$, if f_a is to occur.

(L) $$\frac{\omega L_1}{R_1} = Q \text{ must be } > \frac{2\,C_t}{C_1}$$, if oscillation at antiresonance is to occur.

Since this paper is limited largely to a discussion of plated crystals, it may be stated that a crystal unit has the appearance of a condenser, with silver plates separated by a quartz dielectric. As a matter of fact, except in the neighborhood of resonance, it behaves as a condenser of capacitance Co, given by

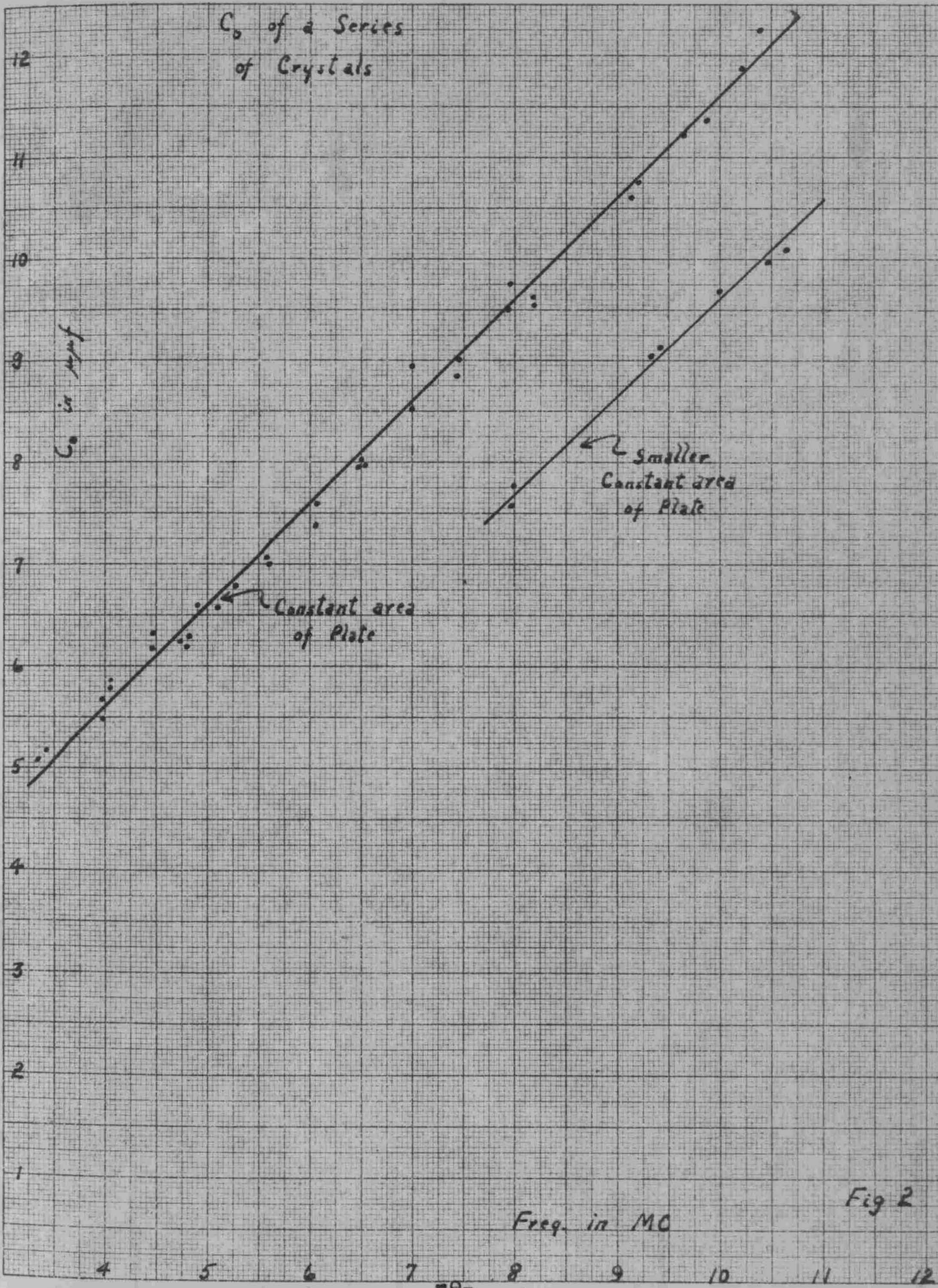
(A). $$Co = \frac{.416\ A\mu\mu f}{t_i}$$

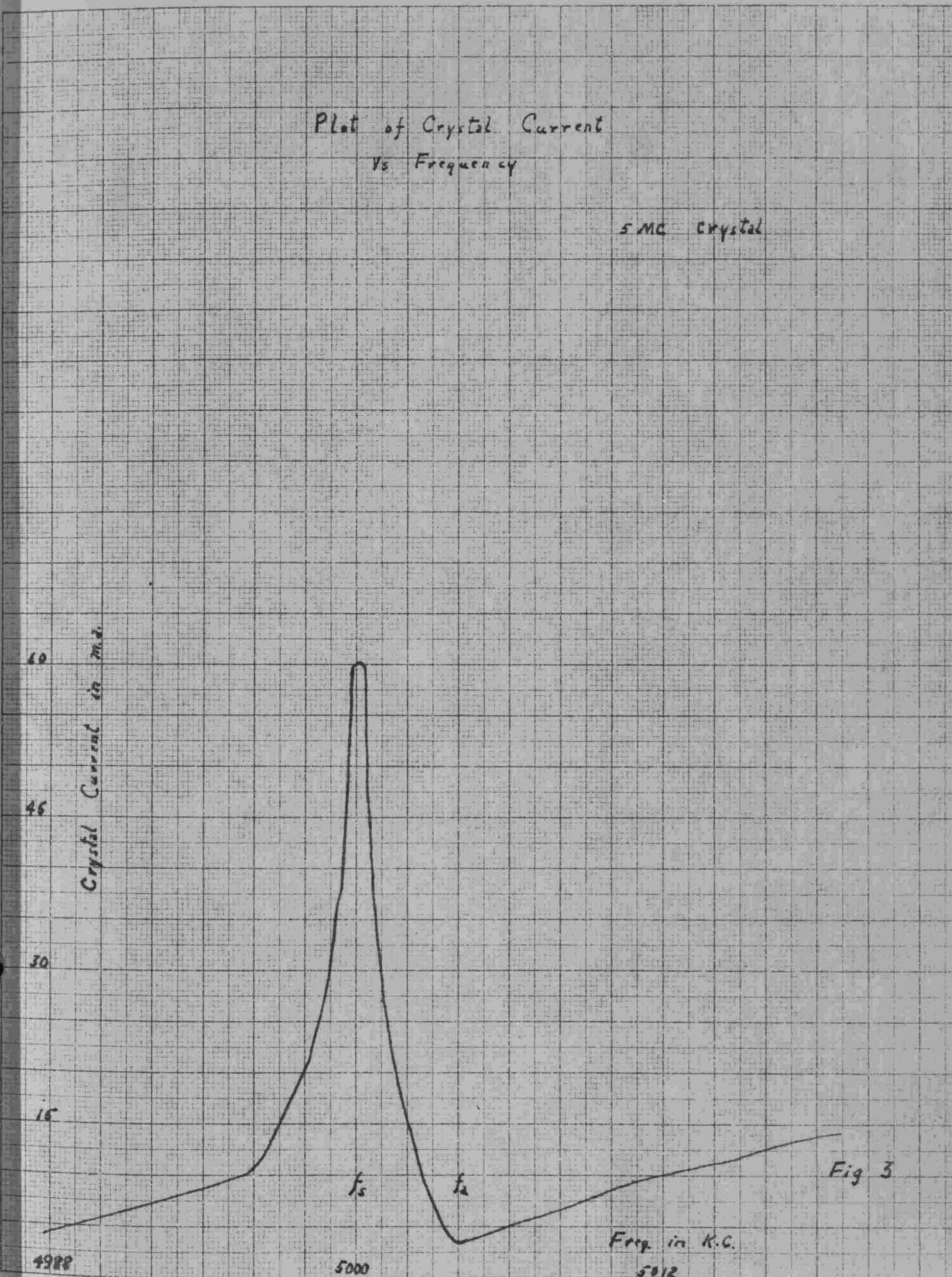
A = area of electrodes, in cm

t_i = thickness of quartz, in cm

Fig. 2 shows that if a series of AT crystals of various frequencies are plated with the same area of plate, the C_o values will increase directly with frequency because their thickness decreases directly with frequency.
Fig. 3 shows that if a curve of crystal current versus frequency is plotted, crystal current increases with frequency at the lower frequencies, which is characteristic behavior for a condenser, but it also shows that the crystal behaves quite differently in the vicinity of resonance. Here it becomes necessary to consider the equivalent circuit of Fig. 1. Fig. 1 could have been considered orginally as the behavior below resonance is simply an indication that an almost negligible amount of current is then going through the L_1 C_1 R_1 branch due to the very high reactance of the very small C_1, and the C_o branch is the controlling one.

In Fig. 4 there is illustrated the nature of the admittances of the branches of the equivalent circuit and the nature of the total admittance. R_1 is neglected for simplicity. It will be seen that there is a frequency at which the admittance is infinite (very high if R_1 is considered) and a somewhat higher frequency at which the admittance is zero (very low if R_1 is considered). These are the frequencies of series resonance and antiresonance respectively. The diagram shows that as C_t is increased, f_a moves closer to f_s. This is a very important point always in evidence in the data of the crystal measurements.


C_0 of a Series of Crystals
C_0 in $\mu\mu f$
Smaller Constant area of Plate
Constant area of Plate
Freq. in MC
Fig 2
12
11
10
9
8
7
6
5
4
3
2
1
4
5
6
7
8
9
10
11
12


Plot of Crystal Current
Vs Frequency
5 MC Crystal
Crystal Current in m.a.
60
45
30
15
f_s
f_a
Freq in K.C.
4988
5000
5012
Fig 3

Admittance Characteristics of a Crystal

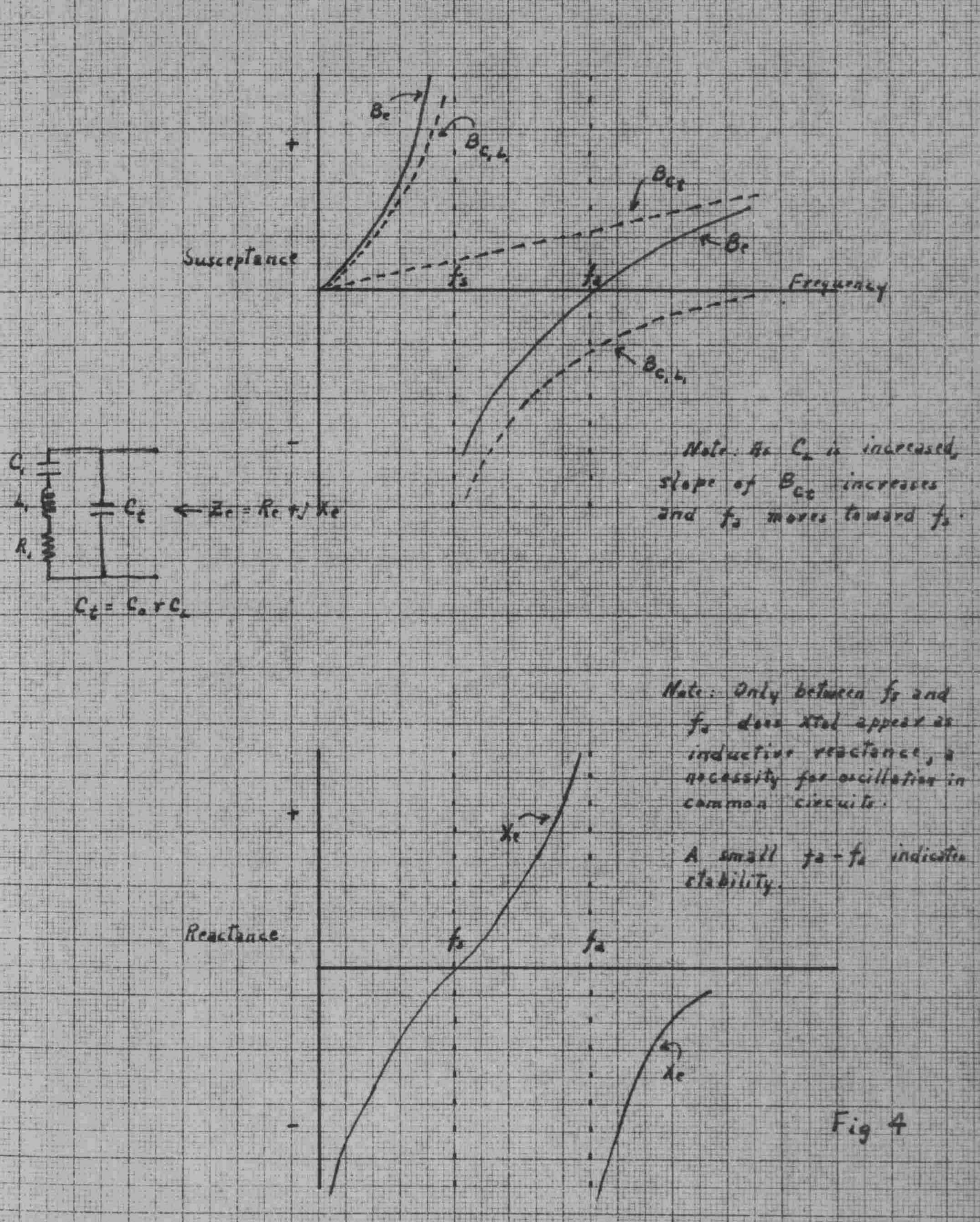


Fig 4

Expressions for f_s and f_a are developed as follows:

Simplifications of the expression for the impedance of the parallel circuit of Fig. 1 gives:

$$Z_e = \frac{R_1 + j\left(\omega L_1 - \frac{1}{\omega C_1} - \omega^3 L_1^2 C_t + 2\omega L_1 C_t/C_1 - C_t/\omega C_1^2 - \omega R_1^2 C_t\right)}{C_t^2/C_1^2 + 1 - 2\omega^2 L_1 C_t + 2\, C_t/C_1 + \omega^2 R_1^2 C_t^2 - 2\omega^2 L_1 C_t^2/C_1 + \omega^4 L_1^2 C_t^2}$$

Setting the imaginary part equal to zero and clearing:

$$\omega^2 L_1^2 C_1^2 - C_1 - \omega^4 L_1^2 C_1^2 C_t + 2\omega^2 L_1 C_1 C_t - C_t - \omega^2 R_1^2 C_1^2 C_t = 0$$

If the final term is neglected, two solutions for ω result; in terms of frequency they are:

(B). $$f_s = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1}} \qquad f_a = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1} + \frac{1}{L_1 C_t}}$$

These are approximate values of series resonance and antiresonance respectively.

If the expression for Z e is broken into its terms, they are:

(C_1) $$R_e = \frac{R_1}{\omega^2 C_t^2\left[R_1^2 + \left(\omega L_1 - \frac{1}{\omega C_1} - \frac{1}{\omega C_t}\right)^2\right]}$$

$$(C_2)\quad X_e = \frac{\left(\omega L_1 - \frac{1}{\omega C_1}\right)\frac{1}{\omega^2 C_t^2} - \left[R_1^2 + \left(\omega L_1 - \frac{1}{\omega C_1}\right)^2\right]\frac{1}{\omega C_t}}{R_1^2 + \left(\omega L_1 - \frac{1}{\omega C_1} - \frac{1}{\omega C_t}\right)^2}$$

These are exact expressions and may be reduced to approximate expressions of interest as follows:

Neglect R_1^2 in denominator of expression for R_e

$$R_e \cong \frac{R_1}{\omega^2 C_t^2\left(\omega L_1 - \frac{1}{\omega C_1} - \frac{1}{\omega C_t}\right)^2}$$

$$R_e \cong \frac{R_1}{\left[\omega^2 C_t L_1 - \frac{C_t}{C_1} - 1\right]^2} \cong \frac{R_1}{\left(\frac{\omega^2 C_t}{\omega_s^2 C_1} - \frac{C_t}{C_1} - 1\right)^2}$$

Since $\omega_s^2 = \frac{1}{L_1 C_1}$ or $L_1 = \frac{1}{\omega_s^2 C_1}$,

let $\Delta\omega = \omega - \omega_s$, an increment in frequency above that of resonance.

$$\frac{\Delta\omega}{\omega_s} = \frac{\omega}{\omega_s} - 1 \qquad \text{or} \qquad \frac{\omega}{\omega_s} = \frac{\Delta\omega}{\omega_s} + 1 .$$

$$\frac{\omega^2}{\omega_s^2} = \left(\frac{\Delta\omega}{\omega_s}\right)^2 + 2\,\frac{\Delta\omega}{\omega_s} + 1$$

$$R_e \cong \frac{R_1}{\left\{\left[\left(\frac{\Delta\omega}{\omega_s}\right)^2 + 2\frac{\Delta\omega}{\omega_s} + 1\right]\frac{C_t}{C_1} - \frac{C_t}{C_1} - 1\right\}^2}$$

$$R_e \cong \frac{R_1}{\left\{\frac{\Delta\omega^2}{\omega_s^2}\frac{C_t}{C_1} + 2\frac{\Delta\omega}{\omega_s}\frac{C_t}{C_1} - 1\right\}^2} = \frac{R_1}{\left\{1 - 2\frac{\Delta\omega}{\omega_s}\frac{C_t}{C_1}\right\}^2}$$

Neglecting $\frac{\Delta\omega^2}{\omega_s^2}$ term,

(D1) $R_e \cong \frac{R_1}{\left\{1 - 2\frac{C_t}{C_1}\frac{\Delta f}{f_s}\right\}^2}$ in vicinity of resonance; i.e. when $\Delta\omega$ is small.

In the expression for X_e, neglect R_1^2 as before.

$$X_e \cong -\left\{\frac{\omega L_1 - \frac{1}{\omega C_1}}{\omega C_t\left(\omega L_1 - \frac{1}{\omega C_1} - \frac{1}{\omega C_t}\right)}\right\}$$

Above it was seen that $\omega C_t\left\{\omega L_1 - \frac{1}{\omega C_1} - \frac{1}{\omega C_t}\right\}$ proved to be equal to $\frac{2\ \Delta f\ C_t}{f_s} - 1$

$$X_e \cong -\left\{\frac{\omega L_1 - 1/\omega C_1}{\frac{2\Delta f}{f_s}\frac{C_t}{C_1} - 1}\right\} = \frac{\omega L_1 - 1/\omega C_1}{1 - \frac{2\Delta f\ C_t}{f_s C_1}}$$

Since $C_1 = \frac{1}{\omega_s^2 L_1}$, $\quad X_e \cong \frac{L_1\left(\omega - \omega_s^2/\omega\right)}{1 - \frac{2\Delta f\ C_t}{f_s\ C_1}}$

Since $\frac{\Delta\omega}{\omega} = \frac{\omega}{\omega} - \frac{\omega_s}{\omega}$ $\qquad \frac{\omega_s}{\omega} = 1 - \frac{\Delta\omega}{\omega}$

$$X_e \cong \frac{L_1\left[\omega - \omega_s\left(1 - \frac{\Delta\omega}{\omega}\right)\right]}{1 - \frac{2C_t}{C_1}\frac{\Delta f}{f_s}}$$

$$X_e \cong \frac{L_1\left[\omega - \omega_s + \frac{\omega_s}{\omega}\Delta\omega\right]}{1 - 2\frac{C_t}{C_1}\frac{\Delta f}{f_s}} = \frac{L_1\left[\omega - \omega_s + \left(1 - \frac{\Delta\omega}{\omega}\right)\Delta\omega\right]}{1 - 2\frac{C_t}{C_1}\frac{\Delta f}{f_s}}$$

neglecting $\frac{\Delta\omega^2}{\omega}$ in numerator,

$$(D_2) \qquad X_e \cong \frac{2\,\Delta\omega\, L_1}{1 - \frac{2C_t\,\Delta f}{C_1\, f_s}} \cong \frac{4\pi\, L_1\, \Delta f}{1 - \frac{2\,C_t\,\Delta f}{C_1\, f_s}}$$

Consideration of the (D) expressions shows:

1. If $\Delta f = 0$, i.e. at series resonance,

   $R_e \cong R_1$ and $X_e \cong 0$

   or $Z_e \cong R_1$ at series resonance.

2. (D_2) shows that if Δf is negative, i.e. below series resonance X_e will be negative. If Δf is positive, i.e. just above series resonance, and if Δf is not too large, there is a possibility of X_e being positive. Just how big it can become before the denominator of the expression for

X_e goes negative, depends on the relative sizes of Ct, C_1, and f_s. Just where f_a occurs depends on these same things. It turns out, just as shown in Fig. 4 that Xe can only be a positive reactance between f_s and f_a.

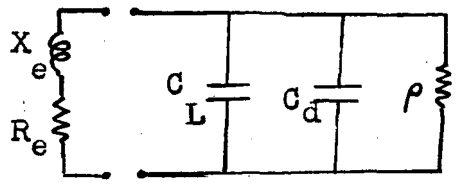
Returning to the original exact expression for Z_e , neglecting the $\omega R^2 C_t$ term and substitution of

$$\omega_a = \sqrt{\frac{1}{L_1 C_1} + \frac{1}{L_1 C_t}} \quad \text{gives}$$

$$(E_1) \quad Z_e \text{ at } f_a \cong \frac{1}{\omega_a^2 C_t^2 R_1}.$$

This impedance at antiresonance is called the Performance Index or PI of the crystal. An instrument for the measurement of PI of crystals in described by C. W. Harrison (3) of the Bell Telephone Laboratories. The P.I. figure has been proposed as a measure of crystal merit, based on the following generally reasoning:

Oscillators of the Pierce or Miller type may be represented thus:



Re $+$ j^{x}e = Effective impedance presented by crystal at its oscillating frequency

C_L = input capacity measured at terminals of circuit across which crystal is connected. (Tube cold)

C_d = dynamic capacity contributed by tube circuit.

ρ = negative resistance which is characteristic of an "amplifier" input impedance when the plate load is inductive.

The general theory is that any oscillator which operates the crystal at an antiresonant frequency, or in its positive reactance region, appears to the crystal as a capacitive reactance and negative resistance paralleled across the terminals to which the crystal is connected. The combination of crystal and tube circuit operate at a frequency such that the total reactance is zero and at an amplitude such that the total resistance is zero.

For clarity, it should be mentioned that at higher frequencies (say 50MC.) it is likely that the crystal will be utilized as a series element in the feedback path of the oscillator, comprising a "series - resonance" oscillator, rather than the "positive reactance" or "antiresonance" type.

This P.I. will be of interest in a consideration of what constitutes a satisfactory C_o for crystals, which is taken up in a later chapter. It should be observed that while the expression for P.I., indicates superficially that the lower C_o gives a better P.I., a lower Co goes hand in hand with a <u>larger</u> R_1, which has a tendency to reduce P.I.

Another interesting approximate relation can be developed between R_e and CL or C_o
from (D2), letting $C_t = C_o$, i.e., considering the crystal alone,

$$\frac{R_e}{R_1} \cong \frac{1}{\left[1 - \frac{2\,C_o}{C_1} \quad \frac{\Delta f}{f_s}\right]^2}$$

From(D_2)

$$X_e \cong \frac{4\pi L_1 \, \Delta f}{1 - 2\,\frac{C_o}{C_1} \quad \frac{\Delta f}{f_s}}$$

At oscillating frequency $X_e \cong X_{c_L}$

$$\therefore \quad 1 - \frac{2\,C_o}{C_1} \quad \frac{\Delta f}{f_s} \cong \frac{4\pi L_1 \, \Delta f}{X_{CL}} = (4\, L_1 \, \Delta f)\,(2\pi f\, C_L)$$

$$\therefore \quad \frac{R_e}{R_1} = \left[\frac{1}{(2\pi f\, C_L)\,(4\pi L_1 \, \Delta f)} \right]^2$$

Now $\quad f_s^{\,2} = \dfrac{1}{4\pi^2 \, L_1 \, C_1} \quad$ or $\quad 4\pi L_1 = \dfrac{1}{\pi f_s^{\,2} C_1}$

Also $\quad \omega_s^2 = \dfrac{1}{L_1 \, C_1} \qquad \omega_a^2 = \dfrac{1}{L_1 C_1} + \dfrac{1}{L_1 \, C_t}$

$$\omega_a^2 - \omega_s^2 = \frac{1}{L_1 \, C_t}$$

$$(\omega_a + \omega_s) \quad (\omega_a - \omega_s) = \frac{1}{L_1 C_t}$$

$$(\omega_a + \omega_s) \quad (\Delta\omega) = \frac{1}{L_1 \, C_t} = \frac{C_1}{L_1 C_1 \, C_t} = \frac{\omega_s^2 \, C_1}{C_o + C_L}$$

$$\Delta\omega = \frac{\omega_s^2 \, C_1}{(\omega_a + \omega_s) \quad (C_o + C_L)}$$

(E2) $$\Delta f \cong \frac{f_s^2 \; C_1}{(f_a + f_s)(C_o + C_L)} \cong \frac{f_s \; C_1}{2\,(C_o + C_L)}$$

Substituting for $4\pi L_1$ and Δf

$$\frac{R_e}{R_1} \cong \left[\frac{1}{(2\pi f\, C_L)\left\{\frac{1}{\pi f_s^2 C_1}\right\}\frac{f_s^2 C_1}{(f_a+f_s)\,(C_o+C_L)}}\right]^2 \cong \left[\frac{1}{2fC_L \;\frac{1}{(f_a+f_s)(C_o+C_L)}}\right]^2$$

$$\cong \left[\frac{C_o + C_L}{C_L}\right]^2$$

(F). or $$R_e \cong R_1 \left(1 + \frac{Co}{C_L}\right)^2$$

This shows that as CL increases, R_e decreases, and as C_o increases, R_e would appear to increase, but R_1 also decreases, so an optimum value of C_o will exist.

An attempt to show the relation between X_e and C_L or C_o results in the following:

$$X_e \cong \frac{4\pi L_1 \quad \Delta f}{1-\frac{2\,C_o}{C_1}\,\frac{\Delta f}{f_s}} \qquad \frac{R_e}{R_1} \cong \frac{1}{\left[1-\frac{2C_o}{C_1}\;\frac{\Delta f}{f_s}\right]^2} \cong \left\{1+\frac{C_o}{C_L}\right\}^2$$

$$\frac{1}{1-\frac{2\,C_o}{C_1}\;\frac{\Delta f}{f_s}} \cong 1+\frac{C_o}{C_L} \;.\; 1-\frac{2C_o}{C_1}\frac{\Delta f}{f_s} = \frac{C_L}{C_L+C_o}$$

$$4\pi L_1 \Delta f \cong \frac{1}{f_s^2 \pi C_1} \times \frac{f_s^2 \; C_1}{(f_a+f_s)(C_o+C_L)} \cong \frac{1}{\pi (f_a+f_s)\,(C_o+C_L)}$$

$$\therefore X_e \cong \frac{1}{\pi\,(f_a+f_s)\,(C_o+C_L)} \times \frac{C_o + C_L}{C_L} \cong \frac{1}{\pi(f_a+f_s)C_L} \cong \frac{1}{2\pi \; f \; C_L}$$

This simply shows that in the oscillating condition, when the crystal appears as $Z_e = R_e + jX_e$, the crystal will operate at a frequency such that the inductive reactance X_e is approximately equal to the capacitive reactance of the load capacity into which it works.

It will be seen that a measurement of f_s, f_a and C_o of a crystal gives sufficient information to calculate L_1 and C_1.

R, may be found by a substitution measurement method.

Since $$\omega_a^2 - \omega_s^2 = \frac{1}{L_1 C_t}$$

$$(\omega_a - \omega_s)(\omega_a + \omega_s) = \frac{1}{L_1 C_t}$$

$$(2\omega)(\Delta\omega) \cong \frac{1}{L_1 C_t}$$

$$(G_1) \qquad L_1 \cong \frac{1}{8\pi^2 f \, \Delta f \, C_t}\,.$$

C_1 may be calculated from $\omega_s^2 = \dfrac{1}{L_1 C_1}$ or developed as was L_1, so

$$(G_2) \qquad C_1 \cong \frac{2\,\Delta f \quad C_o}{f}$$

The Q of the equivalent circuit of a crystal is sometimes defined as $\frac{\omega L_1}{R_1}$. From (G_1)

(H)
$$Q = \frac{1}{4\pi R_1 C_o \Delta f}$$

A figure concerning the equivalent circuit parameters which is often discussed is the ratio of $\frac{C_o}{C_1}$;

using (G_2)

(I).
$$\gamma = \frac{C_o}{C_1} = \frac{f}{2\Delta f}$$

It is apparent that if a crystal operates at an anti-resonant frequency, since $\omega_a^2 = \frac{1}{L_1 C_1} + \frac{1}{\cancel{L_1(C_o + C_L)}}$, charges in C_L such as might be brought about by changes of tubes or other circuit elements in the oscillator, will cause a change in the operating frequency, or instability.

$$f_a = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1} + \frac{1}{L_1 C_t}}$$

$$\frac{df_a}{dC_t} = -\frac{1}{4\pi L_1 C_t^2\sqrt{\frac{1}{L_1 C_1} + \frac{1}{L_1 C_t}}} \cong \frac{\sqrt{L_1 C_1}}{4\pi L_1 C_t^2}$$

(J). $$\frac{df_a}{dC_t} \cong \frac{-f C_1}{2 C_t^2} = \frac{-f C_1}{2(C_o + C_L)^2}$$

This indicates that change in operating frequency as a result of change in C_L will be minimized if a large C_L is used, which seems reasonable. It superficially indicates that increasing C_o would increase stability, but here again a slight increase in C_o causes a marked increase in C_1 which takes over and actually causes poorer stability with larger C_o. Chapter II shows that an increase in C_o from an average value of 4.1 uuf to 7.6 uuf caused a corresponding change in C_1 from .018uuf to .037uuf.

There are certain combinations of values of L_1 C_1 R_1 and C_o and C_L that represent a crystal that would not operate at an antiresonant frequency. Reference to the admittance curves of Fig. 4, shows for example that if $C_t = C_o + C_L$ were too large, the slope of B_{ct} would be so great that at no point above f_s could $B_{C_t} = B_{C_1L_1}$, giving an f_a. C_1 and C_o cannot be just any size. If all other parameters are of the usual magnitudes, normally C_o or $C_o + C_L$ must be many times greater than C_1 , if antiresonance is to occur. Although it is not shown in Fig. 4, which neglected R_1 , the analysis of Chapter VI shows that R_1 also has its limitations. Examples of relations showing some of the limitations on the circuit parameters are given below:

Consideration of the quadratic in ω^2, a cleared version of the imaginary part of the expression for Z_e from which (B) was derived, indicates that f_a would merge into f_s, i.e. the two roots of the equation will be equal when the discriminant is equal to zero. Setting the discriminant equal to zero,

$$(R_1^4 C_1^4 - 4L_1 R_1^2 C_1^3)C_t^2 + (2R_1^2 L_1 C_1^4)C_t + L_1^2 C_1^4 = 0$$

$$C_t = \frac{-R_1 L_1 C_1 \mp 2 L_1^{3/2} C_1^{1/2}}{R_1^3 C_1 - 4L_1 R_1} \cong - \frac{+ 2 L_1^{3/2} C_1^{1/2}}{4 L_1 R_1}$$

$$(K_1) \qquad C_t \text{ max} \cong \frac{\sqrt{L_1 C_1}}{2 R_1} \cong \frac{1}{2 \omega R_1} = \frac{1}{4 \pi f R_1}$$

It has already been pointed out that as C_L is increased, f_a moves in the direction of f_s. The expression for C_t shows how great C_t can become before f_a and f_s become coincident. If an oscillator is set up with a certain C_t, there is a limit on R_1 given by

$$(K_2) \qquad R_1 \text{ max} = \frac{1}{4 \pi f C_t}$$

From $C_t < \frac{1}{2 \omega R_1}$, Since $\frac{1}{2 \omega R_1} = \frac{\omega L_1}{2 \omega^2 R_1 L_1}$

$$= \frac{Q}{2 \omega^2 L_1} = \frac{QC_1}{2 \omega^2 L_1 C_1} = \frac{QC_1}{2}.$$

C_t must be $< \dfrac{Q\,C_1}{2}$

or Q must be $< \dfrac{1}{2}\,\dfrac{C_1}{C_t}$

(L). $\dfrac{\omega L_1}{R_1} = Q$ must be $> \dfrac{2\,C_t}{C_1}$

III MEASUREMENT OF ELECTRICAL EQUIVALENTS AT LOW, MEDIUM, AND HIGH FREQUENCIES

As was pointed out in the derivations of Chapter II, many things can be calculated concerning the crystal if the following data can be taken:

a. Frequency of series resonance, f_s.

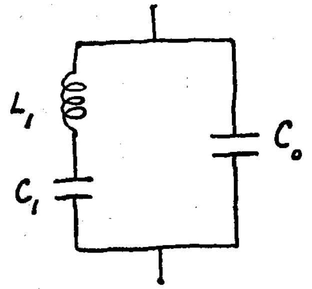
b. Frequency of antiresonance, f_a, using known values of C_L.

c. Values of inter-pin capacity, C_o.

Due to the nature of the circuits used, it was usually convenient to find f_a by finding the frequency at which the crystal in series with C_L reached series resonance, rather than finding the frequency at which the crystal in parallel with C_L reached antiresonance. That this is the same frequency is demonstrated:

Series Resonance

fig.(a)



$$\frac{(X_{L1} + XC_1)(XC_o)}{X_{L1} + XC_1 + XC_o} = 0$$

$$(X_{L1} + XC_1)(XC_o) = 0$$

$$X_{L1} = -XC_1$$

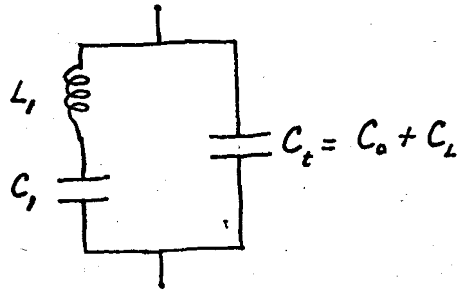
$$\omega L_1 = \frac{1}{\omega C_1}$$

$$\omega_s^2 = \frac{1}{L_1 C_1}$$

$$f_s = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1}}$$

fig.(b)

Antiresonance



$$X_{L1} + X_{C1} + X\,C_t = 0$$

$$\omega L_1 = \frac{1}{\omega C_1} + \frac{1}{\omega C_t}$$

$$\omega_a^2 = \frac{1}{L_1 C_1} + \frac{1}{L_1 C_t}$$

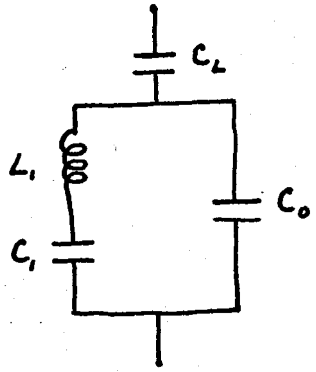
$$f_a = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1} + \frac{1}{L_1 C_t}}$$

$$f_a = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1} + \frac{1}{L_1 (C_o + C_L)}}$$

$$f_a = \frac{1}{2\pi}\sqrt{\frac{1}{L_1 C_1}\,\frac{C_1 + C_o + C_L}{C_o + C_L}}$$

Series Resonance

fig.(c)



$$\frac{(X_{L1}+X_{C1})\,(X_{Co})}{X_{L1}+X_{C1}+X_{Co}}+X_{CL}=0$$

$$X_{L1}X_{Co}+X_{C1}X_{Co}+X_{CL}(X_{L1}+X_{C1}+X_{Co})=0$$

$$-\frac{\omega L_1}{\omega C_o}+\frac{1}{\omega^2 C_1 C_o}-\frac{1}{\omega C_L}\left\{\omega L_1-\frac{1}{\omega C_1}-\frac{1}{\omega C_o}\right\}=0$$

$$-\frac{L_1}{C_o}-\frac{1}{\omega^2 C_1 C_o}-\frac{L_1}{C_L}+\frac{1}{\omega^2 C_L C_1}+\frac{1}{\omega^2 C_L C_o}=0$$

$$-\omega^2 L_1 C_1 C_L+CL-\omega^2 L_1 C_1 C_o+C_o+C_1=0$$

$$\omega^2 L_1 C_1(C_L+C_o)=C_1+C_o+C_L$$

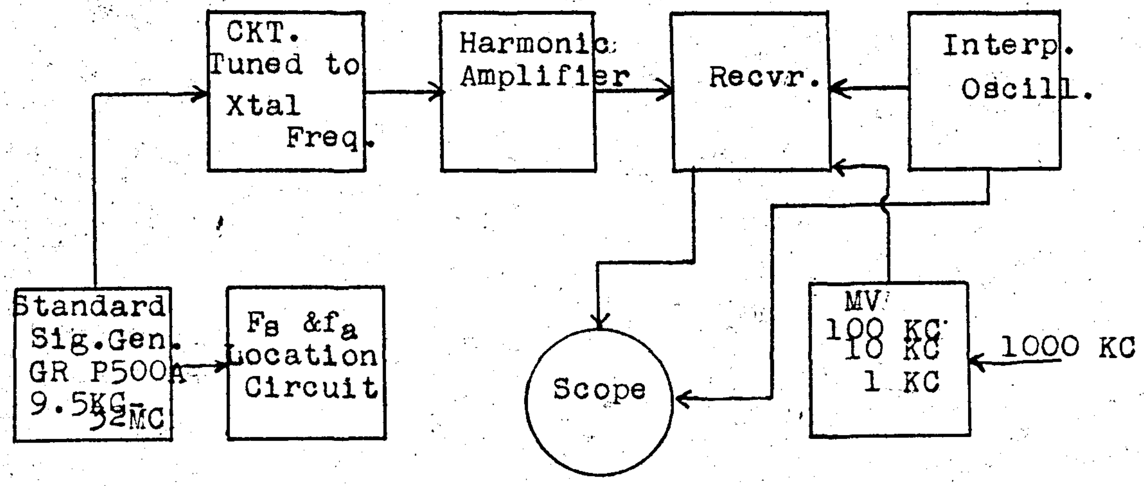
$$\omega^2=\frac{1}{L_1C_1}\,\frac{C_1+C_o+C_L}{C_o+C_L}$$

$$f_s=\frac{1}{2\pi}\sqrt{\frac{1}{L_1C_1}\,\frac{C_1+C_o+C_L}{C_o+C_L}}$$

The above shows that series resonance with the load capacity in series with the crystal occurs at the same point at which antiresonance occurs when the load capacity is in parallel with the crystal. Measurements to locate antiresonance are actually made by finding series resonance with C_L in series with the crystal.

Each of the circuit diagrams carries brief remarks to show their use.

LOW FREQUENCY MEASUREMENT

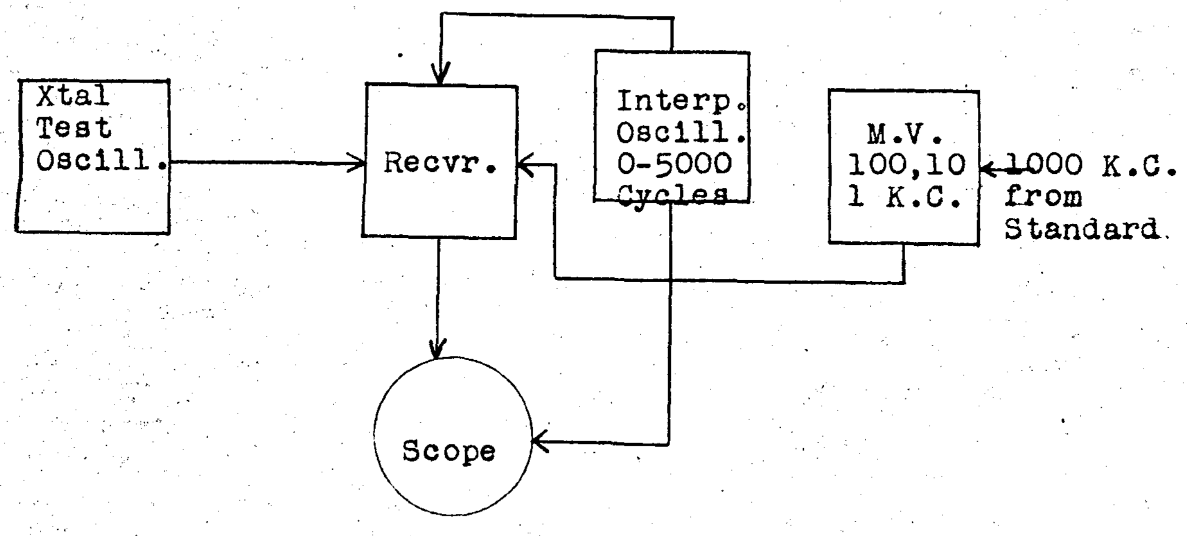


The technique here is to locate f_s and f_a as points of minimum and maximum impedance respectively, and then measure the frequency of the signal generator; to get accuracy the signal is built up by use of a tuned circuit, then put through a harmonic amplifier. A harmonic (on order of fiftieth) is then measured in the usual way. As the frequency of the signal generator is known with a good degree of accuracy, no difficulty is experienced in knowing which harmonic is being used.

The circuit diagram of the f_s and f_a location circuit is given on a separate drawing.

Fig. 5

MEDIUM FREQUENCY MEASUREMENT

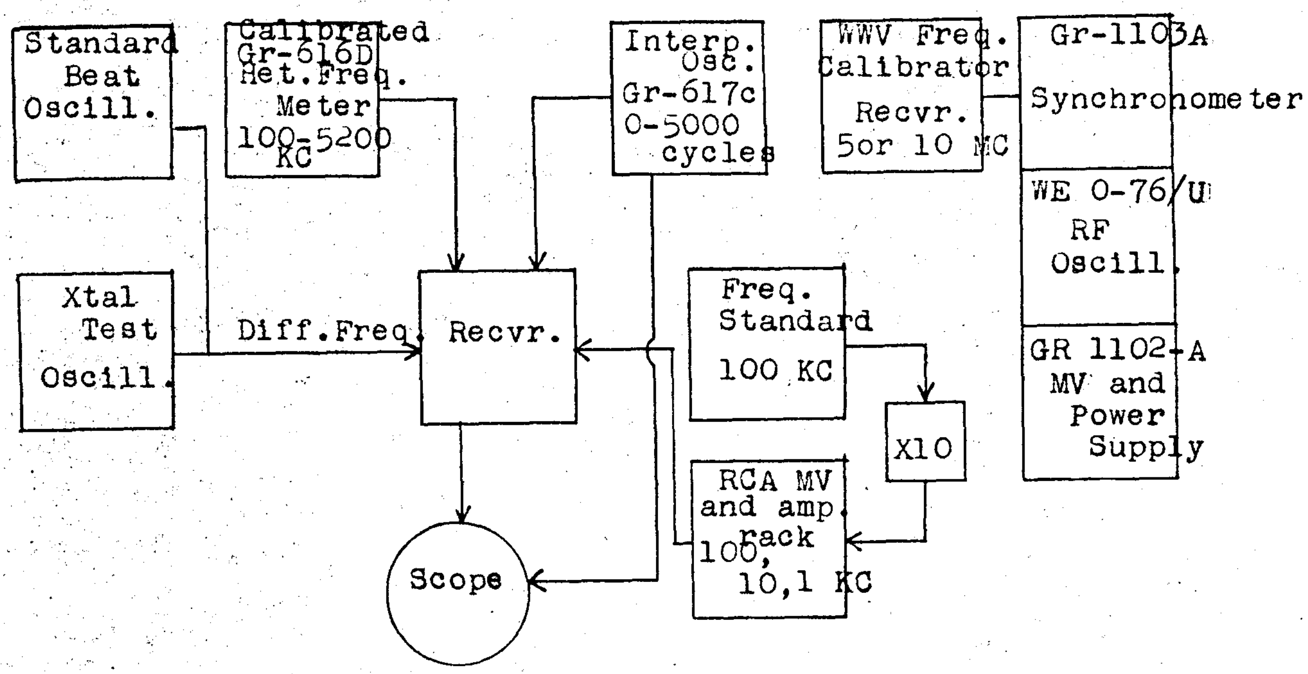


In the region near 10 M.C., frequency may be measured directly with good accuracy. If the frequency of the crystal is not already known approximately some difficulty may be experienced in determining whether the beat frequency is above or below the known harmonic. This can sometimes be done by increasing the capacity across the crystal by grasping the crystal cover or by some similar method. This <u>lowers</u> the frequency of the xtal. If it is beating <u>below</u> a harmonic the beat frequency will raise in pitch; if above, it will decrease in pitch.

Needless to say the 1000 KC synch signal must be stable, as well as the harmonic M.V. The latter may take several hours to settle down and is usually left in a running condition all the time for increased stability.

Fig. 6

HIGH FREQUENCY MEASUREMENT (60 MC).

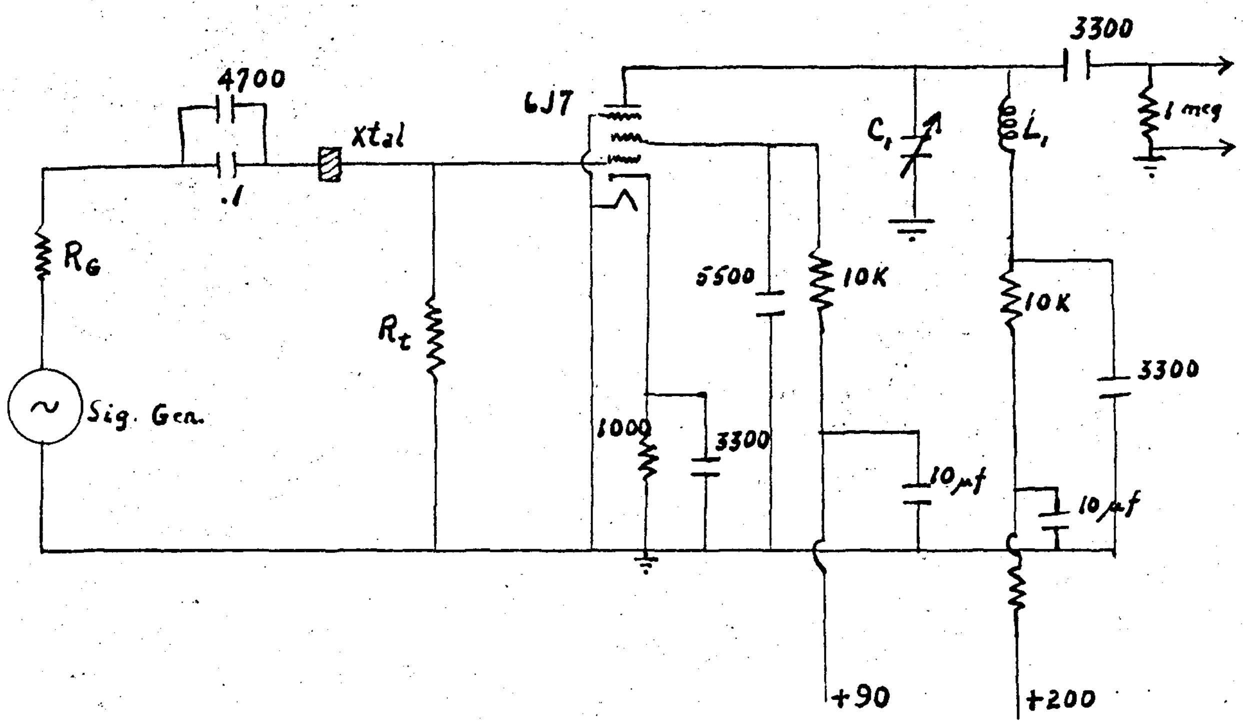


The standard beat oscillator is temperatures controlled and may be 50 MC to beat with a 50-60 MC unknown crystal. If the difference frequency fall within the range of the GR-616-D, it can be read approximately on the calibrated scale of the GR 616-D. This is helpful because of the number of times the frequency is beat. The exact beat frequency is determined in any event by using the GR-617 C. The M.V. rack supplying 100 KC and 10 KC harmonics is synchronized by a 1000 KC signal, coming orginally from the WE-0-76/U oscillator. The WE-0-76/U is checked against WWV by means of the GR-1103A synchronometer.

Fig. 7

CIRCUIT FOR MEASURING ELECTRICAL EQUIVALENTS

AT LOW FREQUENCIES

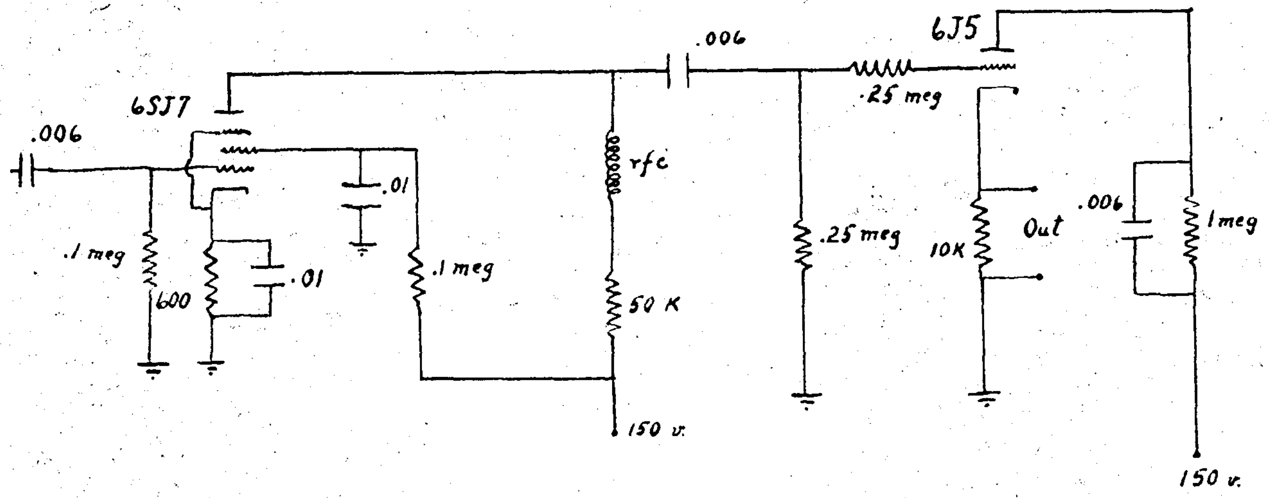


Sig. Generator, G.R. type P-500 A.

R_t, 10.4 ohms used to measure series resonance at 250 KC 1055 ohms used to measure parallel resonance (should be 10% or less of effective resistance of xtal.)

Fig. 8

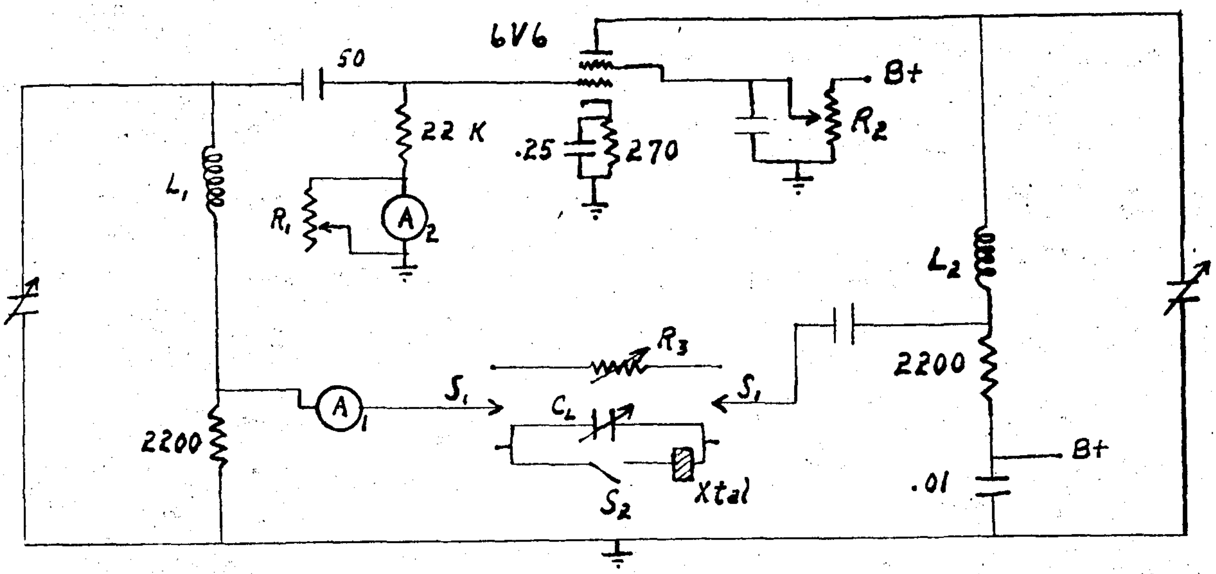
HARMONIC AMPLIFIER USED IN MEASUREMENTS OF LOW FREQUENCIES



In order to secure high accuracy in the measurement of frequencies as low as 100-200 KC, the signal to be measured is routed through a harmonic amplifier such as this and some harmonic (on the order of the fiftieth) measured.

Fig. 9

SIMPLEFIED SCHEMATIC OF MEDIUM FREQUENCY C.I.. METER



R_1 Polentiometer to regulate reading of A2.

R_2 Potentiometer to regulate crystal current.

R_3 Resistors to determine equivalent resistance by substitution.

A_1 0-100 m.a. r.f..meter to measure r.f. current through crystal.

A_2 0-200 uamp DC meter to give a tuning indication.

S_1 Switch to allow substitution of equivalent resistor for crystal.

S_2 Switch to select series operation or simulated anti-resonance operation into variable C_L.

C_L Variable 10-100 uuf simulating reactance into which crystal works

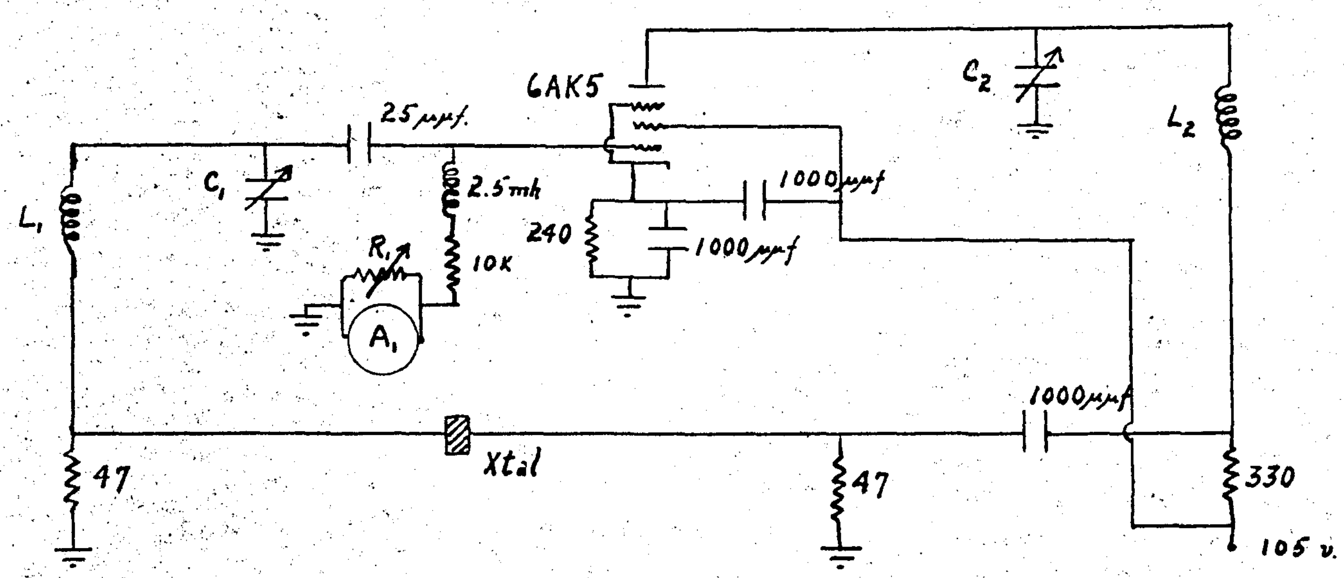
This meter used to measure constants of crystals between 500 and 12,500 K.C. Developed by the Signal Corps, it is called TS-320/TSM

L_1 L_2 Similar pairs of coils selected to cover frequency range:

494-644 KC	1.5mh	1355-2340 KC	.115mh		
638-892 KC	.8mh	2320-4010 KC	39uh	6900-12500 KC	4uh
883-1368 KC	.34mh	3990-6930 KC	13 uh		

Fig..10

SCHEMATIC OF C. I. METER FOR HIGH FREQUENCY CRYSTALS



R_1 Potentiometer to regulate reading of A_1.

A_1 0-200 uamp DC meter to give arbitrary indication of grid current as tuning indication.

C_1 C_2 ganged, each 25 uuf.

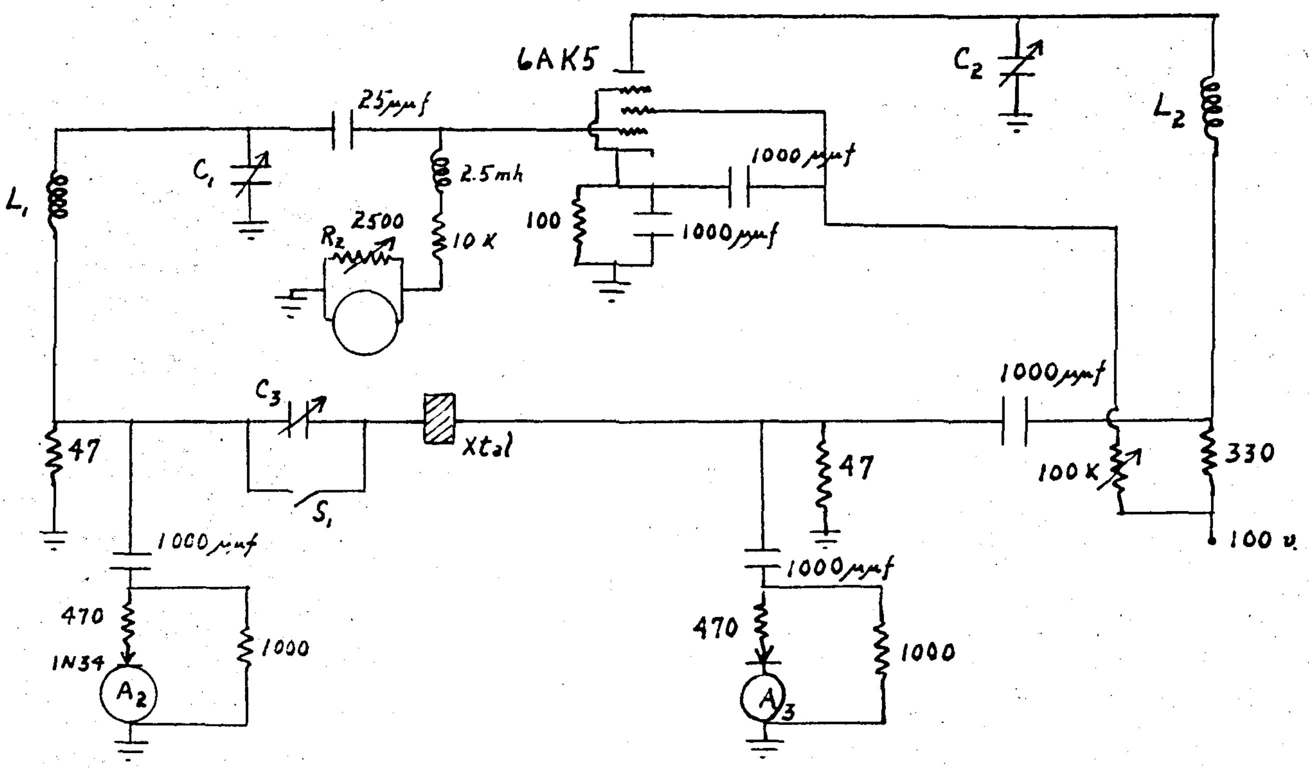
L_1 L_2 Similar pairs of coils selected to cover frequency range:

	Turns	Wire	
9-15MC	25	22	Close wound
14-22	15½	20	" "
22-33	9½	20	" "
32-53	5	20	" "
52-95	3	20	3/8 inch

This meter is used to measure series frequency of high frequency crystals. In a modified form it may be used to measure circuits constants..

Fig. 11

CI METER FOR HIGH FREQUENCY CRYSTALS



Circuit Modified Under Direction of L.L. Dimmick(RCA).

R2 replaced by fixed 2500 ohms.

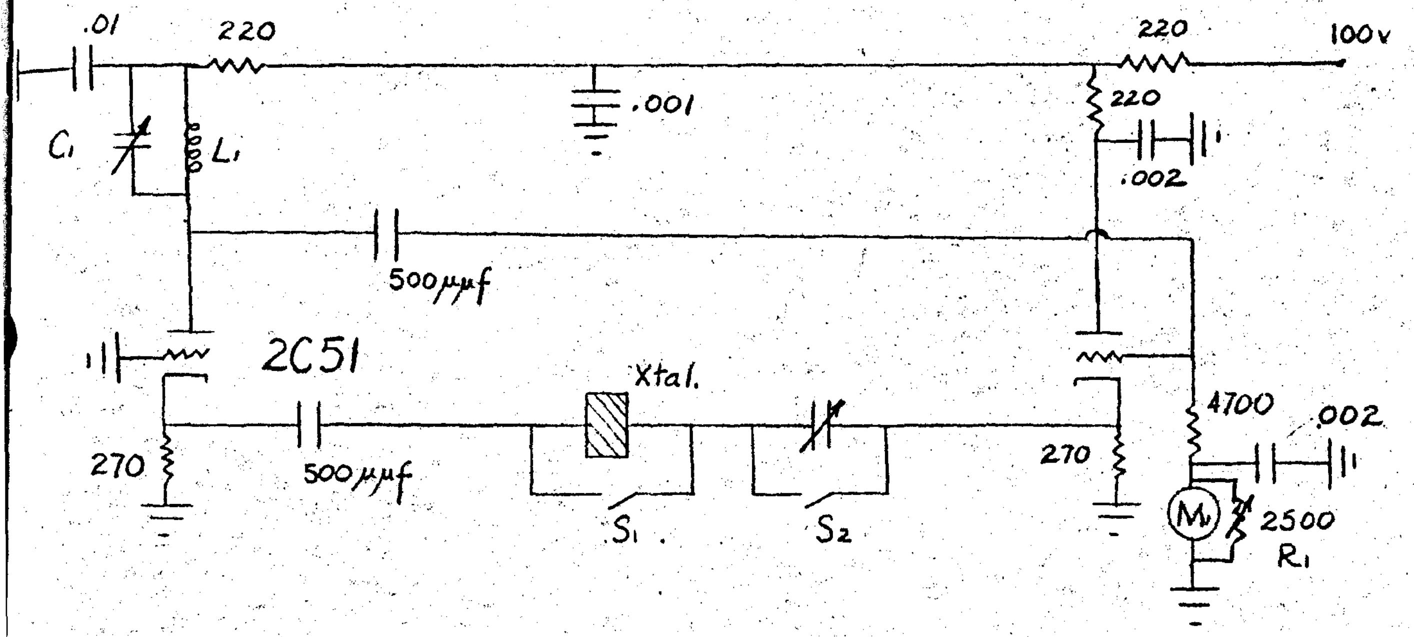
This circuit has additional features as follows:

S_1-C_3 allows operation of crystal in series with standard plug-in condensers of 2 to 92 uuf.

A2 ,A3 allows measurement of voltage across crystal in attempt to locate [redacted] f_s.

Fig. 12

NAVY ZM-2 (XN-1) C. I. METER FOR HIGH FREQUENCY CRYSTALS.

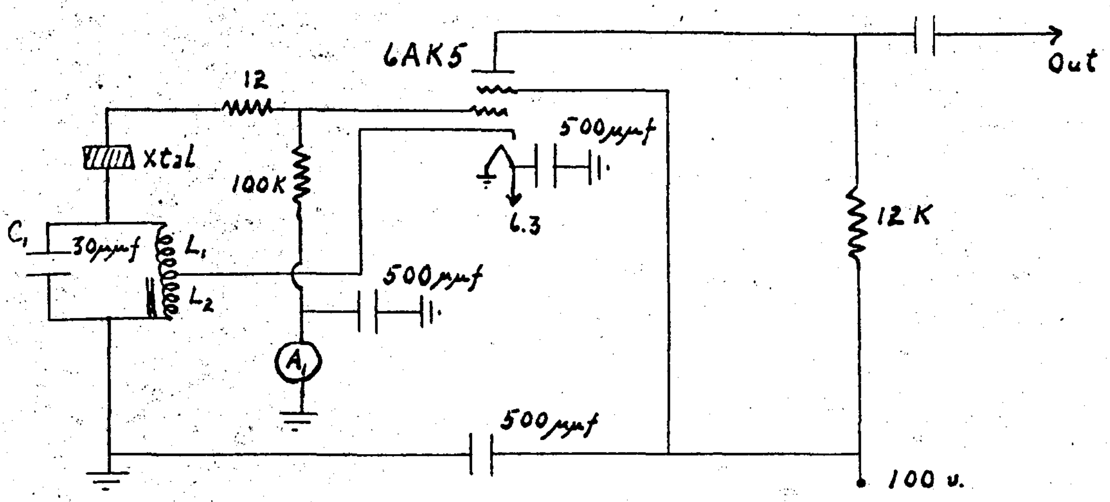


L_1 C_1	Selected to cover frequency of crystal.
S_1	Shorts out crystal to allow setting of R_1 to fix M_1 at arbitrary reading of perhaps 100 uamps. Substitution of say 10 ohms for crystal gives go-no go reading on M_1.
M_1	0-200 uamps.
S_2 C_2	Selects plug-in condensers into which to operate crystal to locate f_a.

Fig. 13

OSCILLATOR CIRCUIT FOR TEMPERATURE CONTROLLED

BEAT FREQUENCY AT 25-50 M.C..



L_1 $L2$ C_1 tune to xtal frequency; permeability tuning of L_2 allows .01% frequency adjust.

A_1 grid current meter.

This stable known output frequency is beat against high unknown frequencies, the difference frequency being easier to measure than the actual unknown.

Fig. 14

IV EFFECT OF C_0 ON EQUIVALENT CIRCUIT PARAMETERS, STABILITY, ACTIVITY

Data concerning the effects of variations of C_0 of medium frequency crystals was gathered by means of measurements upon three groups of crystals.

(1). A group of twelve RCA crystals, of about 10MC, were plated, using successively five different areas of plate, mounted and measured for calculation of equivalent circuit parameters in the CI meter Fig. 10, and checked for stability in the circuits of Fig. 15 and 16.

(2). A group of eighteen Reeves -Hoffman crystals of commercial quality ranging from about 7.8 M.C. to 10.5 M.C., about half of which had C_0 values approximately 2uuf greater than the other half, due to two different areas of plate used in assembling the series.

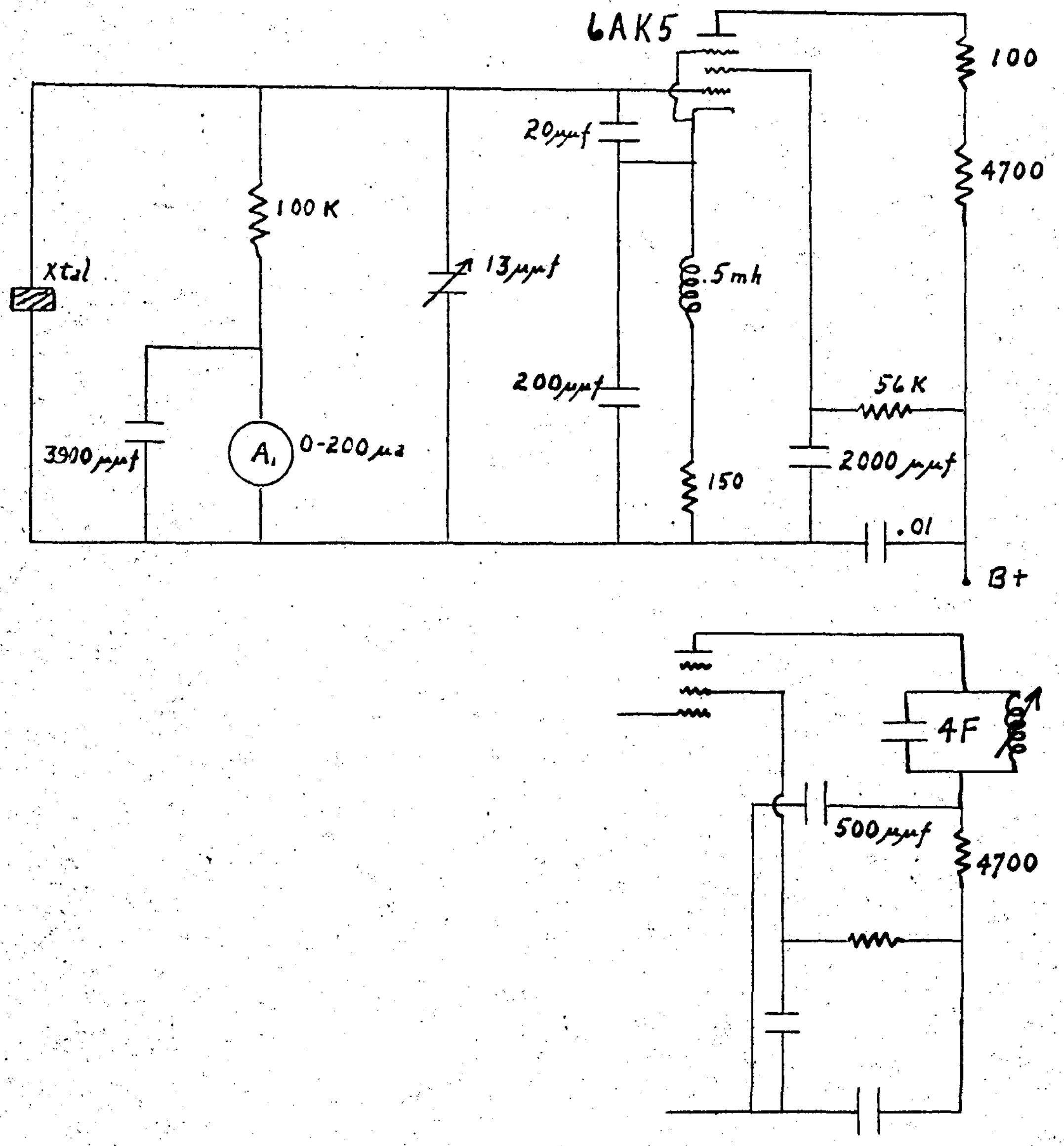
(3). A small group of commercial units, upon which several measurements were made by another experimenter, presented as an example of a different technique in attempting to vary the C_0 of crystal assembled from the same quartz plate.

A discussion of the method of taking the data, with results of each will be taken up, first in relation to stability, and then in relation to activity.

(1). Stability in relation to C_0.

Twelve circular quartz plates of AT cut, with an average unplated frequency of about 9M.C. were selected. During the course of the experiment five circular areas of plate were employed of diameter 140, 173, 212, 250, and 312 mils respectively. These are subsequently referred to as Plates, No.1,2,3,4, and 5; after careful cleaning, all blanks were plated to each of the various area of plate, using the RCA EMV-1 Vacuum Unit (See Fig. 17). The plated crystals were then wire mounted in CR-6 holders, with covers, but not evacuated or gas filled. Measurements were made, using the C. I. meter of Fig. 10, and the circuits of Figs. 15 and 16. After each run, the crystals were disassembled, cleaned, and replated. Some breakage was incurred but most of the data is based upon the results of at least 8 units.

COLLINS OSCILLATOR CIRCUIT



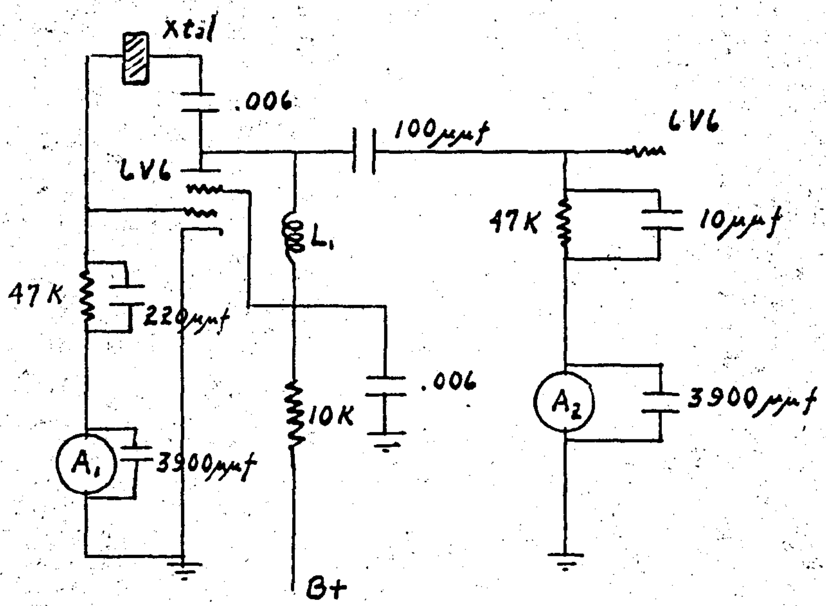
Capacity measured at input terminals of crystals with tubes in, but cold, is .32 uufd. at 4.5 M.C.

Used in procurement of CR-18/U crystals 7375-9100 KC. Load circuit which was orginally tuned to 4F was modified as shown.

Fig. 15

SECTION FROM AVT AIRCRAFT RADIO

TRANSMITTER(RCA) MI-19629



This is from a 3-13 MC 4 channel Xmitter.

A-1 0-200 uamp to give grid to ground voltage.

Plate to ground voltage measured with VTVM.

A-2 0-1000 uamp to give indication of output voltage.

Fig. 16

EMV-1 VACUUM UNIT RCA

This unit has the following features::

6 pairs of electrodes capable of carrying 50 amps each to heat filaments.

2 sputtering electrodes with voltages up to 7 K V mechanical fore pump: cenco-Hypervac 20 (½ hp motor) diffusion pump: vertical oil pump, water cooled.

Pre-pumped to fore-pressure of 50 microns of Hg. (6 minutes) diffusion pumped to .5 to .7 microns of Hg(measured by cold cathode ionization gauge.

The crystals are placed in a masking holder under the BellJar. The Bell Jar is evacuated and then pellets of silver cradled in the heater filaments are caused to boil off silver vapor which is deposited on the desired area of the crystal. Density of plate is controlled by maintaining an ohmic check across a standard glass piece which is plated simultaneously with the crystal until the resistance across it falls to a specified figure..

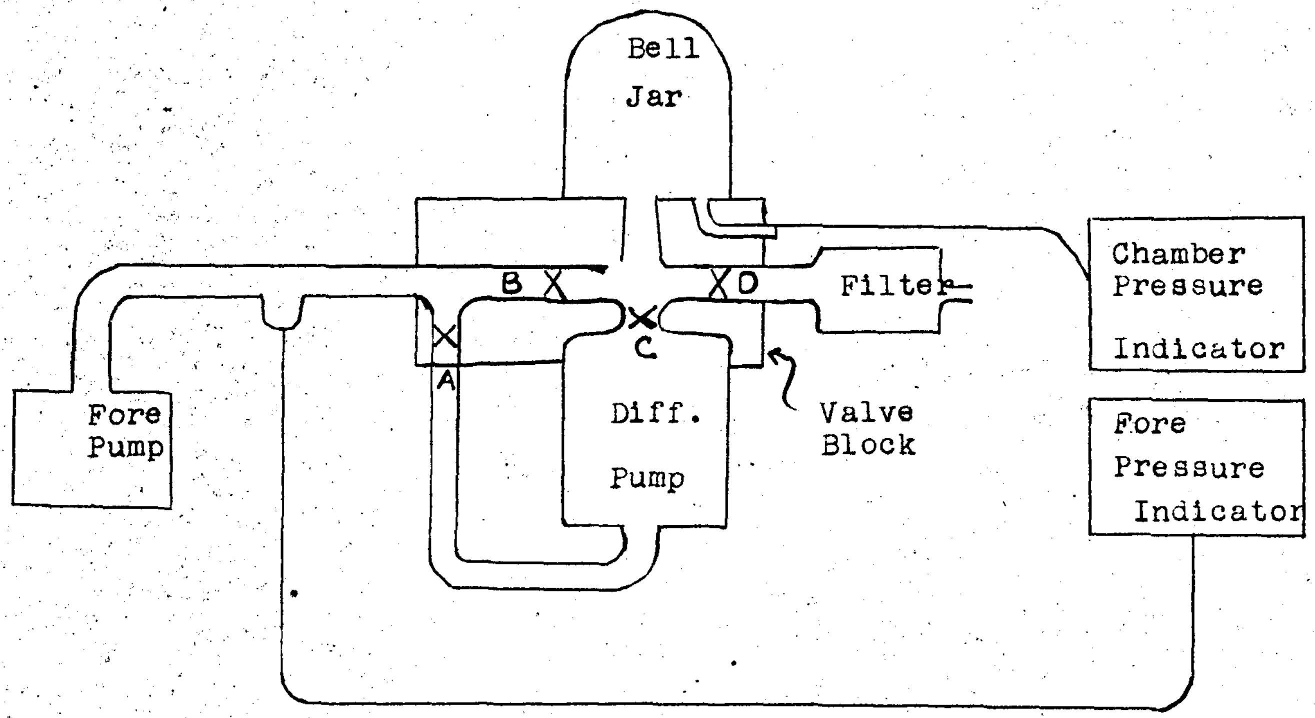


Fig. 17

In the interest of clarity, only certain parts of the data is presented in the text, with complete data given in the appendix.

The effect of silver plating is to reduce the frequency of vibration of a crystal. The greater the area of plating upon the crystal, the greater the reduction in frequency.

Δf in Cycles as Result of Plating

Unit No.	Unplated Freq.M.C.	1.(140mils)	2.(173mils)	3.(212mils)	4.(250mils)	5.(312mils
1	Broken	--------	--------	--------	--------	-------
2	9.031385	--------	-42,371	-58,551	-64,421	-81,552
3	9.037340	--------	--------	--------	--------	-70,661
4	9.032212	-38,141	-39,245	-53,679	-65,865	-75,742
5	Broken	--------	--------	--------	--------	-------
6	9.011648	--------	-37,821	-56,321	-70,231	-70,621
7	9.011000	-38,843	-39,573	-57,523	-71,728	-71,763
8	9.033435	-39,201	-41,271	-53,720	-72,540	-72,679
9	9.008783	-38,901	-38,922	-57,526	-68,161	-68,589
Average C_0 values:		2.7	4.1uuf	5.6uuf	7.6uuf	11.7uuf

This table shows the magnitude of the frequency margin involved in the plating process. Incidentally it shows a factor that must be controlled carefully in the mass production of crystal units to designated frequencies.

The average results of calculations based on all crystals of each area of plate were as follows:

Plate No.	C_o uuf	C_L=32uuf f_a-f_s cycles	L_1h	C_1uuf	R_1ohms	L_1/C_1	P.I.
1	2.7	1892	.0214	.0146	18	1.47 10^{-12}	14,440
2	4.1	2259	.0172	.0181	13.6	.95 10^{-12}	17,600
3	5.6	3090	.0121	.0259	8.6	.47 10^{-12}	25,600
4	7.6	4126	.0087	.0365	7.2	.24 10^{-12}	27,500
5	11.7	5606	.0058	.0547	6.1	.11 10^{-12}	26,700

Why does the frequency decrease when the crystal is silver plated?

$$f_s = \frac{1}{2\pi\sqrt{L_1C_1}}$$

It is seen that if, as the area of plate is increased, L_1 decreased and C_1 changed in the same proportion, the frequency of series resonance would stay the same; C_1 must increase more rapidly than L_1 decreases.

Plate No.	L_1	C_1	L_1 C_1
1	100%	100%	312 10^{-16}
2	80.4	124	312 10^{-16}
3	56.5	177.5	314 10^{-16}
4	40.6	250	317 10^{-16}
5	27.1	374.	317 10^{-16}

The preceding table shows that C_1 does have a tendency to increase slightly faster than L_1 decreases. It must be remembered that this difference <u>will</u> be slight as even a large area of plate causes a decrease in frequency from the unplated frequency of only about 70 K.C. which is only a decrease of about .8% at 9M.C.

There are a number of ways in which to point out the desirability of a low C_o in so far as stability is concerned. In the first place, since a crystal operating at anti-resonance must operate in the region of positive reactance, a small value of fa-fs is itself indicative of stability.

Plate No.	Av.C_o	Av.fa-fs operating into CL = 32uuf
1	2.7uuf	1892 cycles
2	4.1	2259
3	5.6	3090
4	7.6	4126
5	11.7	5602

It would be more informative to have more quantitative results on relative stabilities. All crystals were operated into 32uuf, then 27uuf, and then 37uuf, frequencies with each C_L being measured. Results are presented in tabular form:

Plate No.	Ave. C_o	Av Δf for ΔC_L of 1uuf	Percent	Calculated $\frac{d\,f_a}{d\,C_t}$ based on formula (J)
1	2.7	-------	(.00060%)	54 cycles/uuf
2	4.1	60 cycles	.00067%	63 cycles/uuf
3	5.6	80 cycles	.00089%	83 cycles/uuf
4	7.6	100 cycles	.00111%	105 cycles/uuf
5	11.7	120 cycles	.00130%	129 cycles/uuf

Column 3 is calculated by finding the average Δf_a resulting from a change in C_L of 5 uuf in each case. Column 5 is calculated using:

(J) $\frac{df_a}{dC_t} = \frac{-f\,C_1}{2(C_o + C_L)^2}$, utilizing the average calculated values of C_1 for the various area of plate.

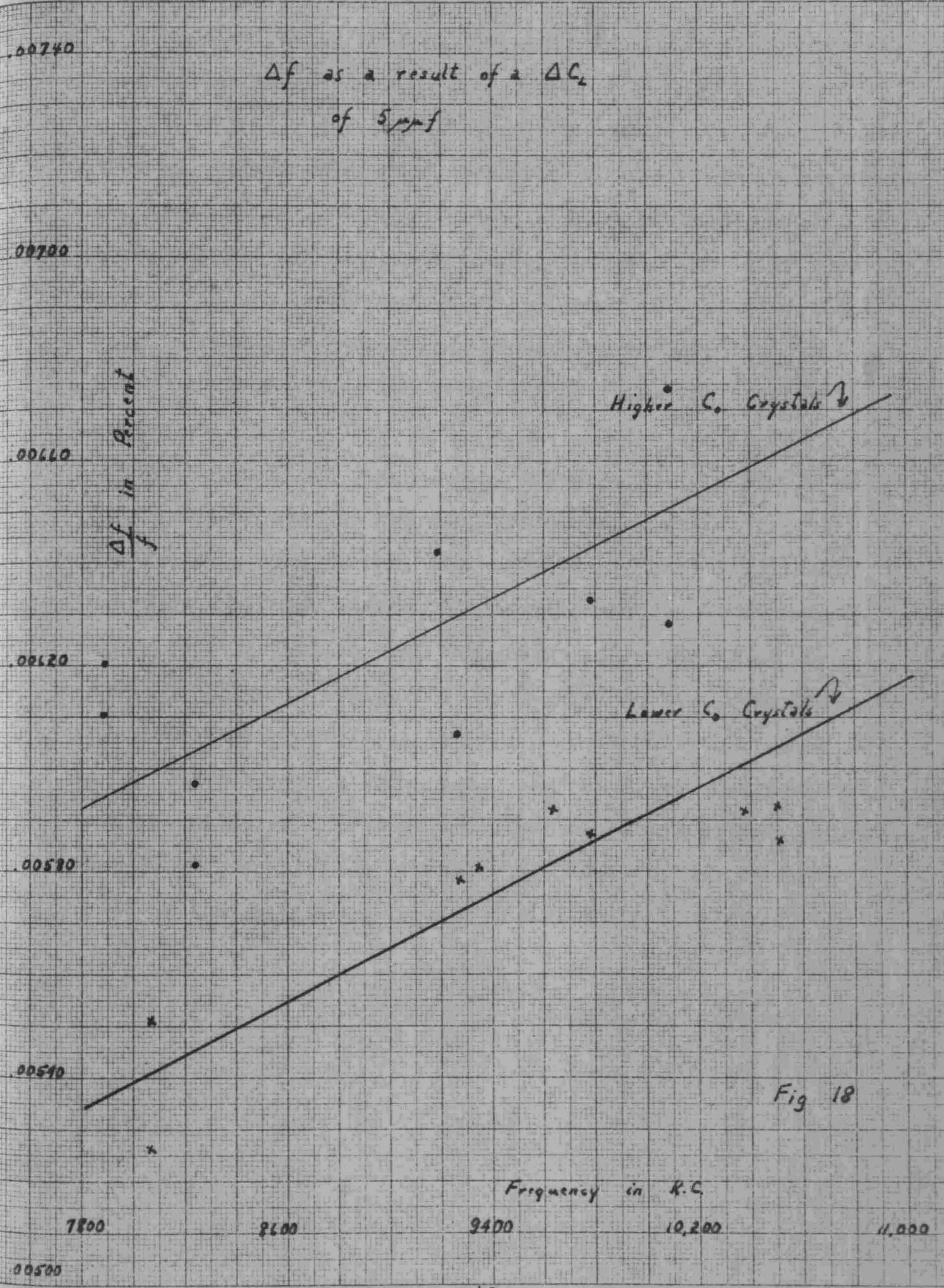
The above results were verified by using 18 Reeves Hoffman crystals of the higher frequencies shown on Fig. 2. These results cannot be tied in directly with the results of the RCA plating experiment. Approximate comparison may be shown as follows, however:

The "Lower C_o" group of R.H. crystals had an average C_o of 9.2uuf. Reference to Figure 18 shows that in the vicinity of 9 M.C., Δf_a as a result of a ΔC_L of 5uuf is about .00562% or a Δf_a as a result of ΔC_L of 1 uuf is about .00112%.

The "Higher C_o" group of R.H. crystals has an average C_o of 10.6uuf. Reference to Fig. 18 shows that in the vicinity of 9M.C., Δf_a as a result of a ΔC_L of 5 uuf is about

.00620% or a Δf_a as result of ΔC_L of luuf is about .00124%.

In view of the fact that the measurements plotted on Fig. 18 are individual frequency measurements where differences between the "High C_o" and the "Low C_o" deviations are of the order of 6 parts in a million exact correlation between these results and those of the RCA experiment would be highly coincidental. It will be seen, however that they do verify the order of magnitude of the percentage of change in f_a as a result of a 1 uuf ΔC_L for various values of C_o.

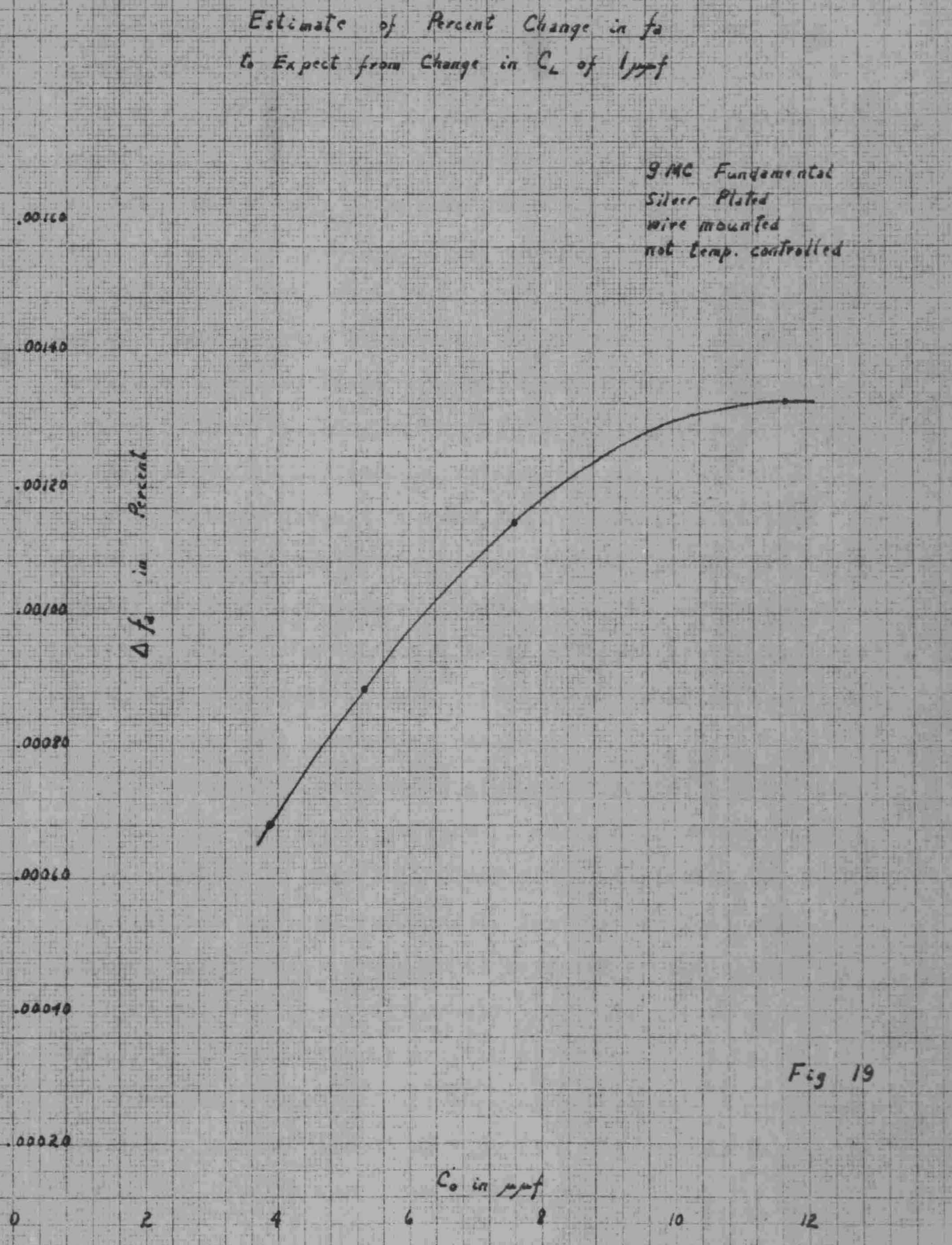


Δf as a result of a ΔC_L
of 5 $\mu\mu f$
Higher C_0 Crystals
Lower C_0 Crystals
$\frac{\Delta f}{f}$ in Percent
.00740
.00700
.00660
.00620
.00580
.00540
.00500
Frequency in K.C.
7800
8600
9400
10,200
11,000

Fig 18

Conclusions as to stability in frequency while operating at antiresonance versus C_0 are presented in Fig. 19.

Of Fig. 19, the following may be summarized: It is likely that an optimum value of C_0 would be selected between 4 uuf and about 12 uuf. Frequency stability requirements will have to be weighed against activity requirements. Activity versus C_0 will be discussed later. The designer of a crystal oscillator will have to determine the maximum anticipated change in input capacity across the terminals to which the crystal is connected. Then he can estimate what this means in terms of change in operating frequency of the crystal. It should be observed that what is now considered rather rigid frequency tolerence on 10MC plated, wire-mounted crystals is of the order of $\pm$ .005%, over a wide temperature range of course. Above, we are dealing with deviations in frequency of the order of .0008 to .0013%. It is likely, at the present stage of crystal manufacture, that aging effects of the crystal itself, its plating, mounts, etc., over a period of a few months will cause as much or more frequency deviation as the deviation that would be caused by changes in CL due to changes in the rest of the oscillator circuit amounting to 1 or 2 uuf.



Estimate of Percent Change in fa
to Expect from Change in C_L of 1μμf
9 MC Fundamental
Silver Plated
wire mounted
not temp. controlled
.00160
.00140
.00120
.00100
.00080
.00060
.00040
.00020
Δ fa in Percent
C₀ in μμf
0
2
4
6
8
10
12

Fig 19

2. Crystal activity.

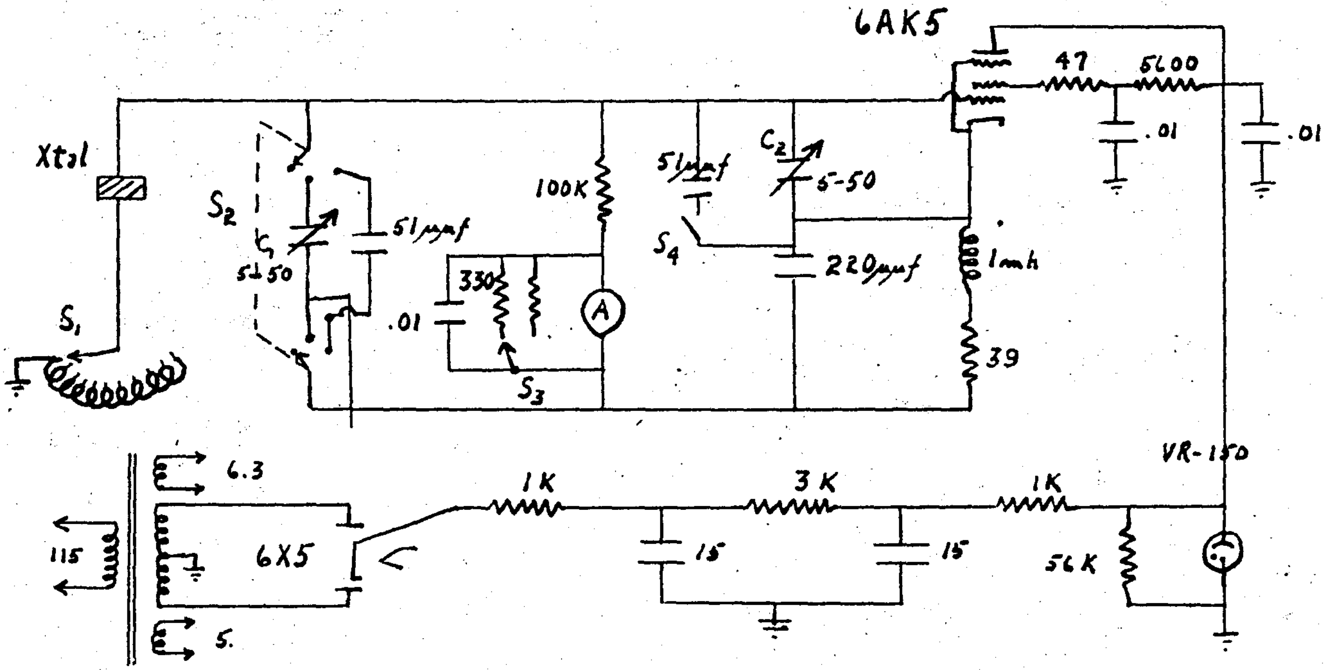
In the past it has been common practice to speak of crystal activity in terms of grid current read in a certain test oscillator circuit. To arrive at a general quantitative comparison of the activity of two groups of crystal, differing say only in their C_o values (this means an attendant resulting difference in values of L_1 C_1 and R_1) seems almost an impossibility. In a particular bridge oscillator one crystal may oscillate and the other not; the one that fails, may oscillate, however, if certain elements of the bridge are changed to suit it. Comparison of the output voltage of the oscillator, using different crystals, may be attempted only to find that the circuit is designed for use at a certain plate voltage and use of any voltage very different from that results in poor performance, regardless of the crystal used. In other words it is difficult to go further than saying that crystals of a certain C_o were tested in a specific circuit and a certain activity indication resulted.

On the subject of various test oscillators, the Collins Radio Company has developed what it calls a Universal Crystal Test Set (shown in Fig. 20). Some of the variables, such as compensations for lead inductance, feedback ratios, shunt capacity across crystal and input capacity of the using oscillators, -these are illustrative of some of the reasons why crystals behave differently in different circuits.

During the course of the plating experiment, the crystals were tested in the circuit of Fig. 15. The circuit was quite stable; the results are shown in Fig. 21.

SCHEMATIC OF COLLINS UNIVERSAL

CRYSTAL TEST SET



		For data taken:
S_1	Compensates for lead inductance in using oscillator. (switching to different crystals, for example)	Set in zero position
S_2	See C_1	Set in open position.
S_3	Meter shunt	Set in open position.
S_4	Allows selection of high feedback ratio encountered in some oscillators.	Set in open position.
C_1	Adjusts shunt capacity across xtal in using circuit.	
C_2	Adjusted to give 32 uufd input.	
A	0-200 uamp.	

Fig. 20

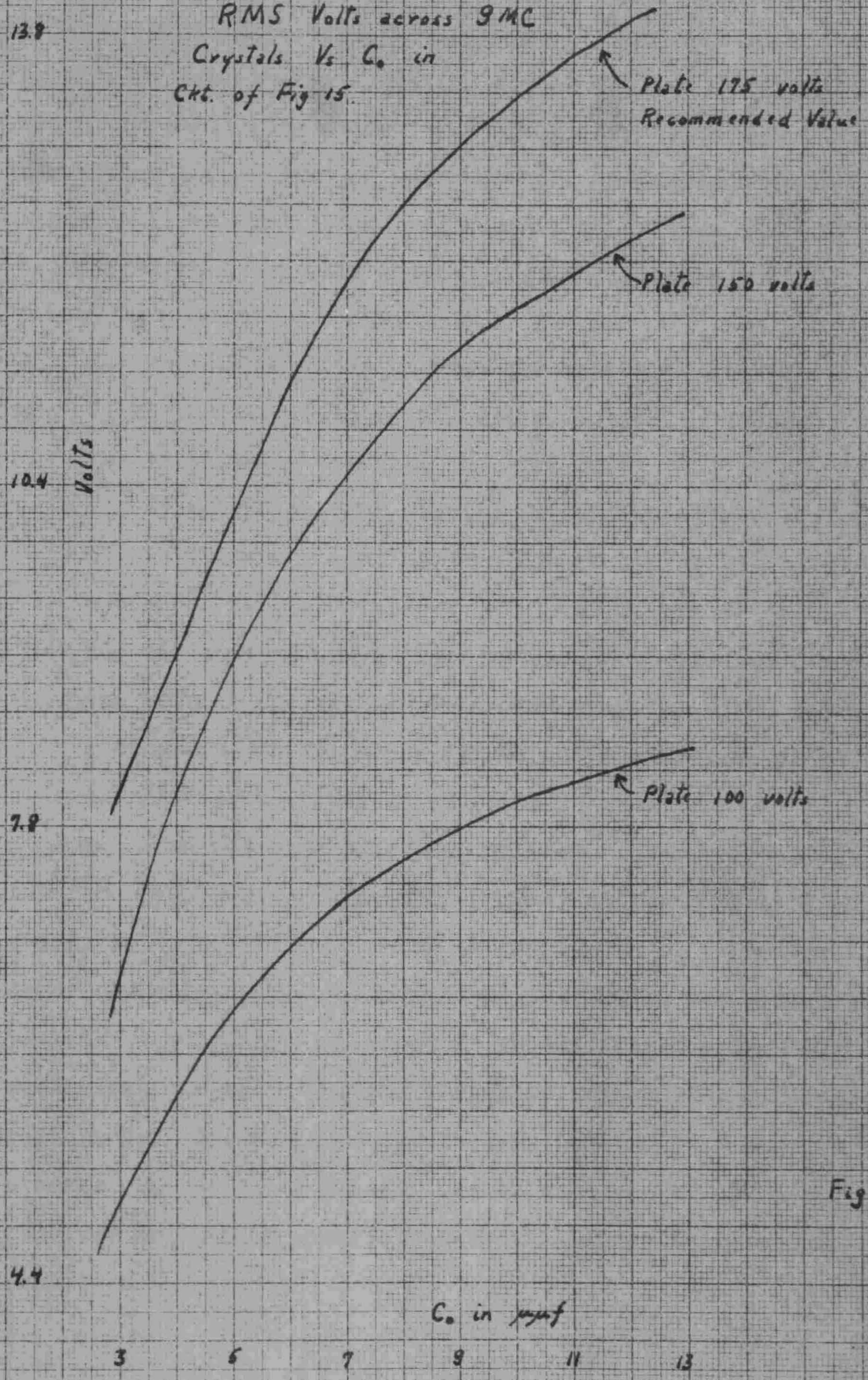
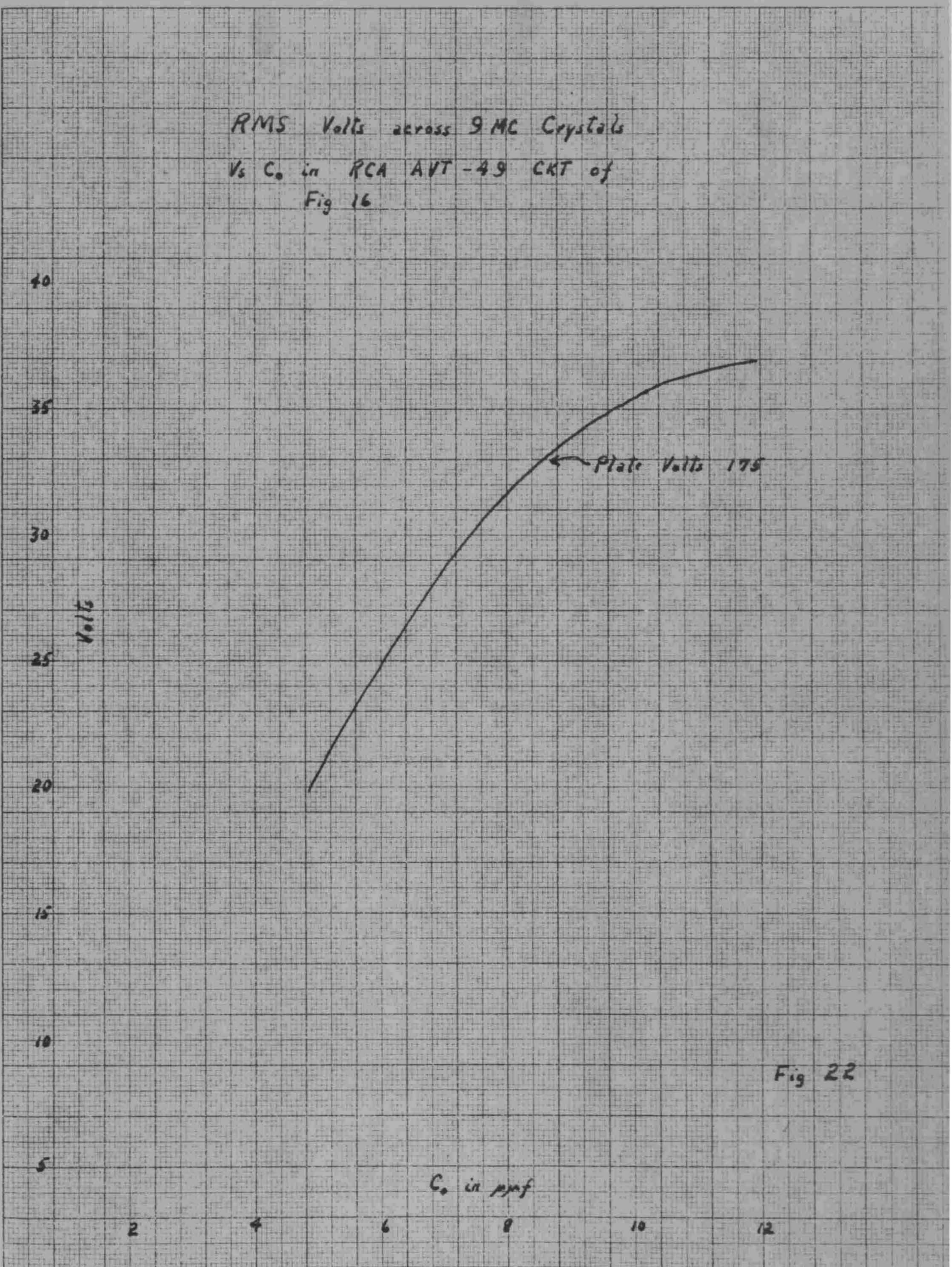

RMS Volts across 9 MC
Crystals Vs C_o in
Ckt. of Fig 15.
Plate 175 volts
Recommended Value
Plate 150 volts
Plate 100 volts
13.8
10.4
7.8
4.4
Volts
C_o in μμf
3
5
7
9
11
13

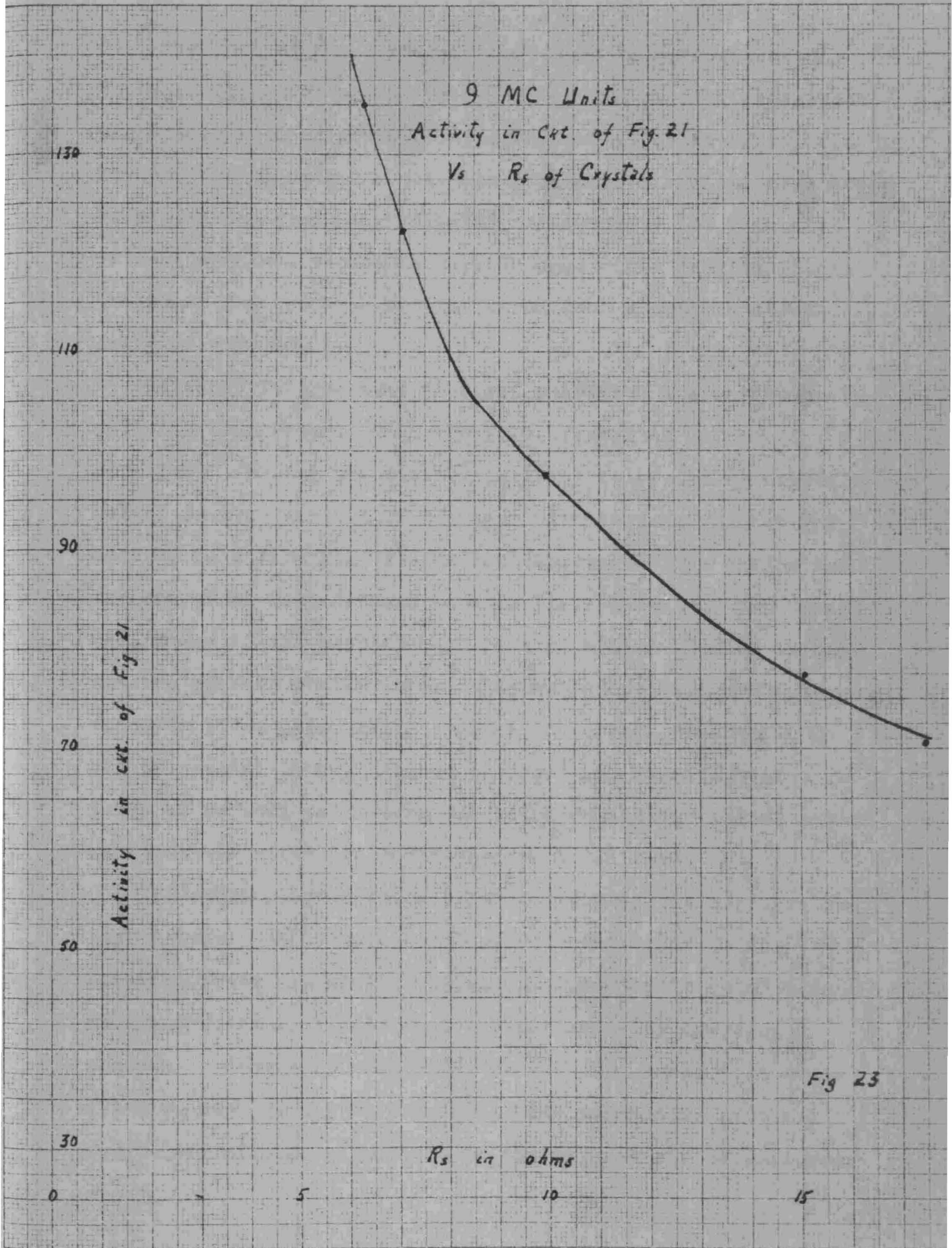
Fig 21

The crystals were also tested in the circuit of Fig. 16 until two crystals were lost due to the setting in of spurious responses that caused excessive voltage across them. It was known that the excitation was excessive but tests were continued for the observation of instability effects until it was feared that too many crystal casualties would result. The following observations were made:

1. The shape of the voltage across the crystal versus C_o curves seemed to verify the results of operation in the more stable circuit. See Fig. 22.
2. Operation was not stable, as evidenced by fluctuations of all voltages measured, but the fluctuations became bad enough to ruin crystals at the lower C_o values (4.1 5.6 and 2.7 uuf). This is apparently due to the greater heating effects induced in the low C_o, high R, crystals, giving rise to frequency drifts and encouraging coupling in of spurious frequencies.

Measurements made by Mr. R. D. Bigler, formerly a Collins Radio Company engineer, now RCA crystal engineer, verify the nature of variation in crystal activity with C_o variation described above. Mr. Bigler was investigating the effective resistance tolerance suggested in recent proposed crystal specifications when they are presented in terms of effective resistance. It should be remembered, however that as C_o increases, R_s and R_e decrease. The results of Fig. 21 are repeated in Fig. 23 except that in the latter they are presented in terms of R_1.


RMS Volts across 9 MC Crystals
Vs C_o in RCA AVT-49 CKT of
Fig 16
Volts
40
35
30
25
20
15
10
5
Plate Volts 175
C_o in $\mu\mu f$
2
4
6
8
10
12
Fig 22


9 MC Units
Activity in Ckt of Fig. 21
Vs Rs of Crystals
130
110
90
70
50
30
Activity in ckt. of Fig 21
Rs in ohms
0
5
10
15
Fig 23

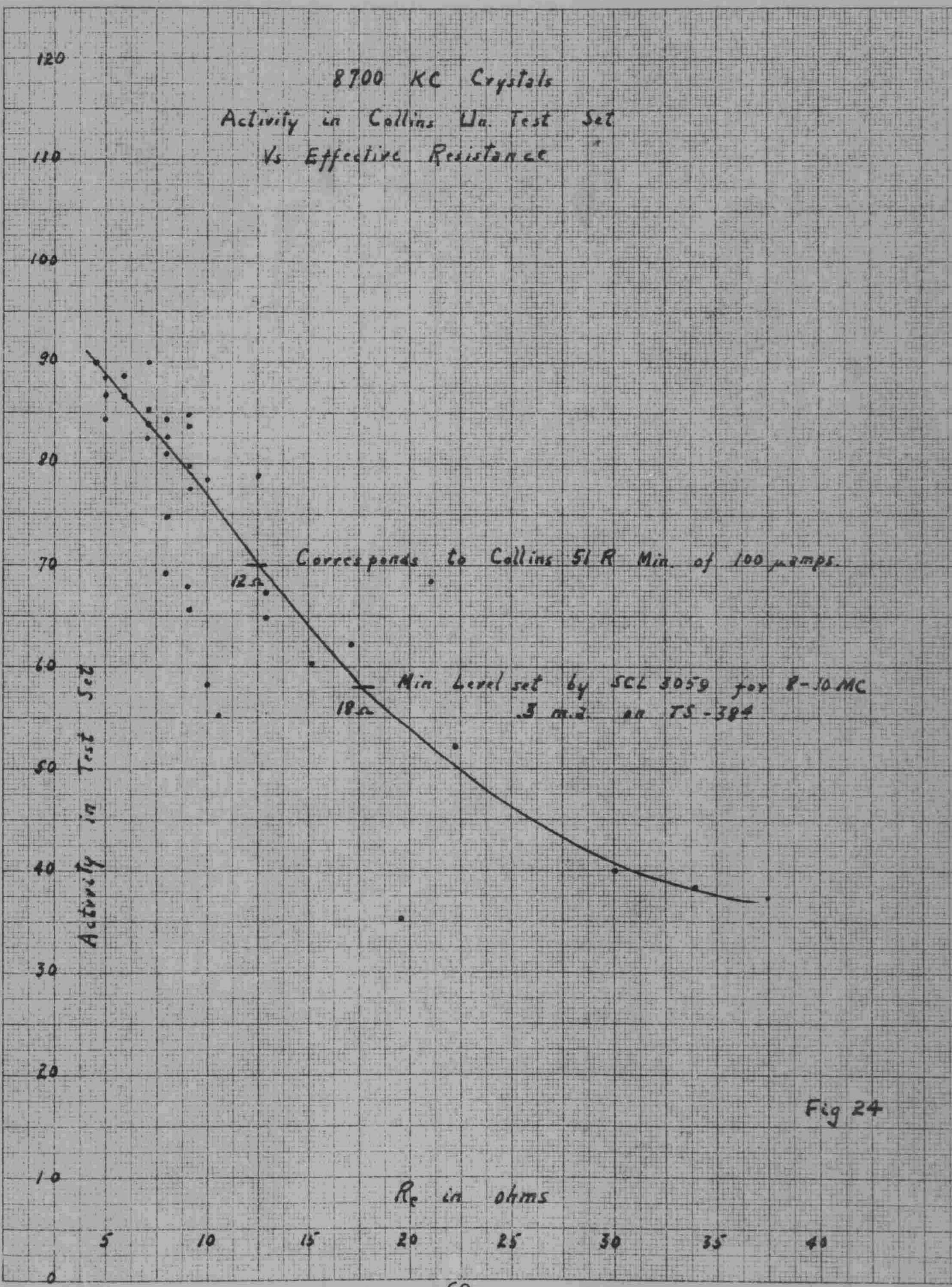
The work of Mr. Bigler is presented to call attention to the alternative method by which the RCA plating experiment might have been conducted; he caused variation in C_0 by using the "stripping method"; starting with crystals with large areas of plate, and gradually reducing the areas by its removal, without complete removal and replating. This is a difficult procedure and accounts for the variations about the smooth curve of Fig. 24.

Fig. 25 is presented to again point out the variation in activity that results with the same crystal in different circuits. Knowing the minimum activity tolerated of crystals tested in the 51-R and TS-384 test circuits, Mr. Bigler was able to read from Fig. 25 the equivalent activity rating in the Universal Test Set and show on Fig. 24 that if the effective resistance of the crystals had been more than 12 ohms in one case or 18 ohms in the other, they would not have met previous specifications under which crystals have been procured. This was of special interest since proposed new specifications in terms of crystal parameters suggested something like 50 ohms as a maximum effective resistance to be allowed.

In summary, concerning crystal activity, it is repeated that general quantitative results, with reference to no particular circuit appears beyond the possibilities of this study; greater C_0 values mean greater activity however. Concerning frequency stability, it has been shown that quantitative results can be obtained. The parameters of the equivalent circuit parameters of the crystal can be measured.

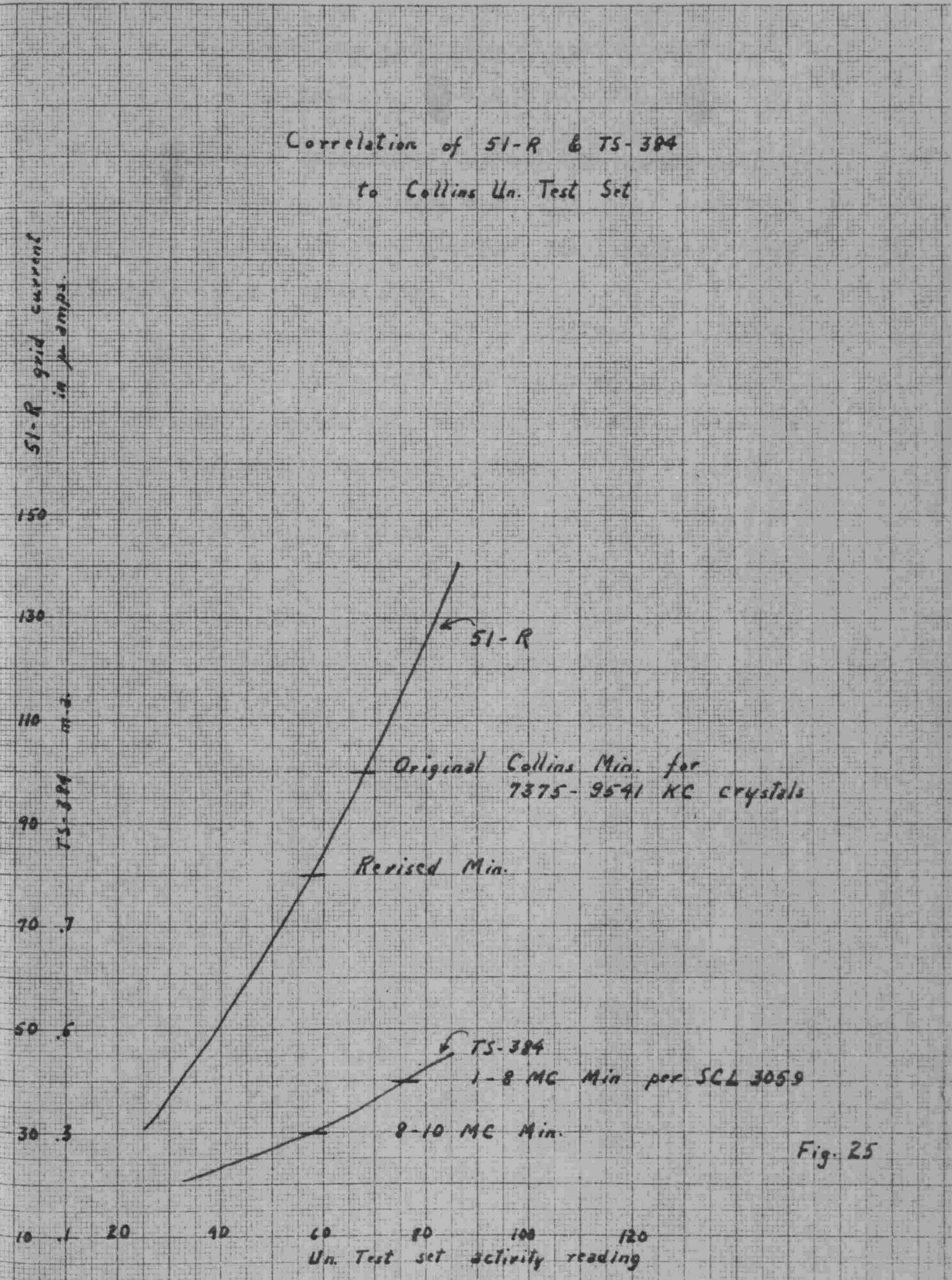
Then, using (J) $\frac{df_a}{dc_t} = \frac{-f\ C_1}{2(C_o + C_L)^2}$

the amount of deviation in operating frequency with the maximum change in C_t anticipated can be calculated. If as much activity, or output, as possible is desired, the maximum C_o, in keeping with the frequency deviation that can be tolerated, should be used.



8700 KC Crystals
Activity in Collins Un. Test Set
Vs Effective Resistance
120
110
100
90
80
70
60
50
40
30
20
10
0
Activity in Test Set
Corresponds to Collins 51 R Min. of 100 μamps.
12Ω
Min Level set by SCL 3059 for 8-30 MC
.3 ma. on TS-384
18Ω
5
10
15
20
25
30
35
40
R_e in ohms

Fig 24



Correlation of 51-R & TS-384
to Collins Un. Test Set
51-R grid current
in μ amps.
TS-384 m.a.
150
130
110
90
70 .7
50 .5
30 .3
10 .1
20
40
60
80
100
120
Un. Test set activity reading
51-R
Original Collins Min. for
7375-9541 KC crystals
Revised Min.
TS-384
1-8 MC Min per SCL 3059
8-10 MC Min.

Fig. 25

V. VARIATIONS OF EQUIVALENT CIRCUIT PARAMETERS AT SAME AND DIFFERENT FREQUENCIES.

RCA (7) has made measurements of the equivalent circuit parameters of crystals in the frequency range from 80-250KC produced by a number of companies. The results of these measurements are as follows:

L_1 is given in henries, C_o in uuf, and R_s in ohms. Unit designations refer to various manufacturers. Crystals are all not temperature controlled and are of various construction designs:

Frequency = 80 KC

Units	L_1	C_o	R_s
RH-1	20.1	45	200
JK-1	47.8	34.5	345
SC-1	33.2	48.8	372
CE-1	34.3	48.5	388
RV-1	52.4	28.2	630
BL-1	54.7	20.3	700
PRL-1	50.4	21.7	800
CL-1	39.9	32.6	1645
CRL-1	96.8	6.3	28000(abnormal)
BL-4	77.6	17	1000
Max.	54.7	48.8	1645
Min.	20.1	17	200

Frequency = 100 KC

Units	L_1	C_o	R_s
MW-153	72.3	6.8	308
RH-3	17.0	32.7	360
SC-2	----	----	---
JK-2	50.8	22.0	510
AM-2	40.0	25	650
PRL-2	37-2	19.0	675
CE-2	----	----	---
RV-2	54.4	19.7	930
BL-5	31.4	25.4	1215
RH-L-79	37.6	21.2	1750
BL-7	72.6	12.8	1840
CRL-2	47.2	23.	2380
EB-1	55.2	21.9	2490
EB-2	46.5	22.6	5720
CL-4	80.0	25.5	1885
RV-6	56.1	16.5	1340
WE-3	67.0	6.3	1030
RV-8	56.7	17.2	1435
RV-9	54.1	18.5	1015
-10	58.6	16.4	2810
-11	53.7	18.8	1100
-12	59.2	16.5	1170
-13	60.4	16.0	915
JK-6			
JK-7	35.4	12.7	910
Max.	80	32.7	5720
Min.	17	6.3	308

Frequency = 150 KC

Unit	L_1	C_0	R_s
MU-758	25.9	16.8	257
PRL-3	32.8	10.6	1085
BL-6	29.7	12.8	1120
SC-3	29.3	16.9	1190
RH-2	18.8	14.1	1245
CE-3	31.0	16.4	1485
JK-3	50.9	11.6	1515
AM-3	26.0	16.6	1765
RV-3	22.6	21.6	2200
BL-2	75.5	7.7	2700
CL-2	47.7	10.4	2990
CRL-3	47.3	11.0	4600
WE-4	43.2	4.5	1630
Max.	75.5	21.6	4600
Min.	18.8	7.7	257

Frequency = 200 KC

Unit	L_1	C_o	R_s
JK-4	9.1	28.9	232
CRL-4	10.4	22.2	710
SC-4	25.8	11.6	1450
RH-5	47.8	6.2	1480
AM-4	30.9	8.1	1831
BL-3	30.7	7.2	1960
PRL-4	31.0	6.4	2060
WE-1	45.8	3.0	2690
CL-3	25.0	11.2	2880
CE-4	30.0	10.5	3100
RV-5	24.6	13.7	3700
BL-10	64.7	6.4	5600
RV-7	24.6	13.7	1140
GE-1	21.7	15.7	1290
Max.	64.7	28.9	5600
Min.	9.1	3.0	232

Frequency = 250 KC

Unit	L_1	C_o	R_s
RV-4	21.1	15.7	4700
PRL-5	33.8	3.9	3600
BL-9	15.7	10.9	1033
BL-8	15.5	11.9	1600
CRL-5	41.6	5.4	6800
RH-4	29.7	6.1	1276
CE-5	25.4	8.4	1128
SC-5	27.2	7.9	1316
JK-5	8.0	19.1	380
CL-5	20.0	9.5	3550
AM-5	25.0	6.3	2392
WE-2	37.6	2.6	2410
Max.	41.6	15.7	6800
Min.	8.0	2.6	380

Observation of these measurements brings out clearly the fact that crystals of different design at any one frequency are likely to vary widely in their parameter values. If the crystals are all of the same design and built by the same company, variations of the following order of magnitude have been observed:

Units	Freq.KC	L_1henries	$C_o$uuf	R_s ohms
BL-6	4165	.058	6	38
-7	4165	.056	6	24
-8	4450	.044	6.5	17
-9	4450	.050	6.2	22
-19	6050	.019	7.3	12
-20	6050	.0195	7.6	13

In general it is observed that:

(1) Measured values of L_1 C_1 R_1 or C_o of crystals of same frequency, but different design may vary as much as several hundred percent.

(2) Measured values of these parameters of crystals of same frequency, same design, can probably be made not to vary more than about 50%. More variation than is indicated in the examples above is often observed.

(3) The order of magnitude of the parameters for the various frequencies can be seen from the data presented.

As a matter of interest the results of illustrative measurements in circuits of Figs. 12 and 13 are given below:

Measuring Ckt.	Unit	Freq.	Δf (cycles)	C_L	C_o	R_s	L_1
Fig. 13	BL-49	60MC.	1300	92uuf	17.7uuf	45	.0015 h
Fig. 12	RCA-148	43.63MC.	1720	33	12.5	38	.0037
Fig. 12	Ma-81	42.35MC.	1760	33	11	--	.00385
Fig. 13	BL-29	60MC.	1235	92	17.3	--	.0016
Fig. 13	RCA-128	25MC.	3612	10	8.	20	.0160

The accuracy of the above results is not claimed to be high, as will be shown in the next chapter.

The National Bureau of Standards has measured the R_1 of a number of crystals at high frequencies but have not been asked to measure the other circuit parameters. Illustrations of their results are given below:

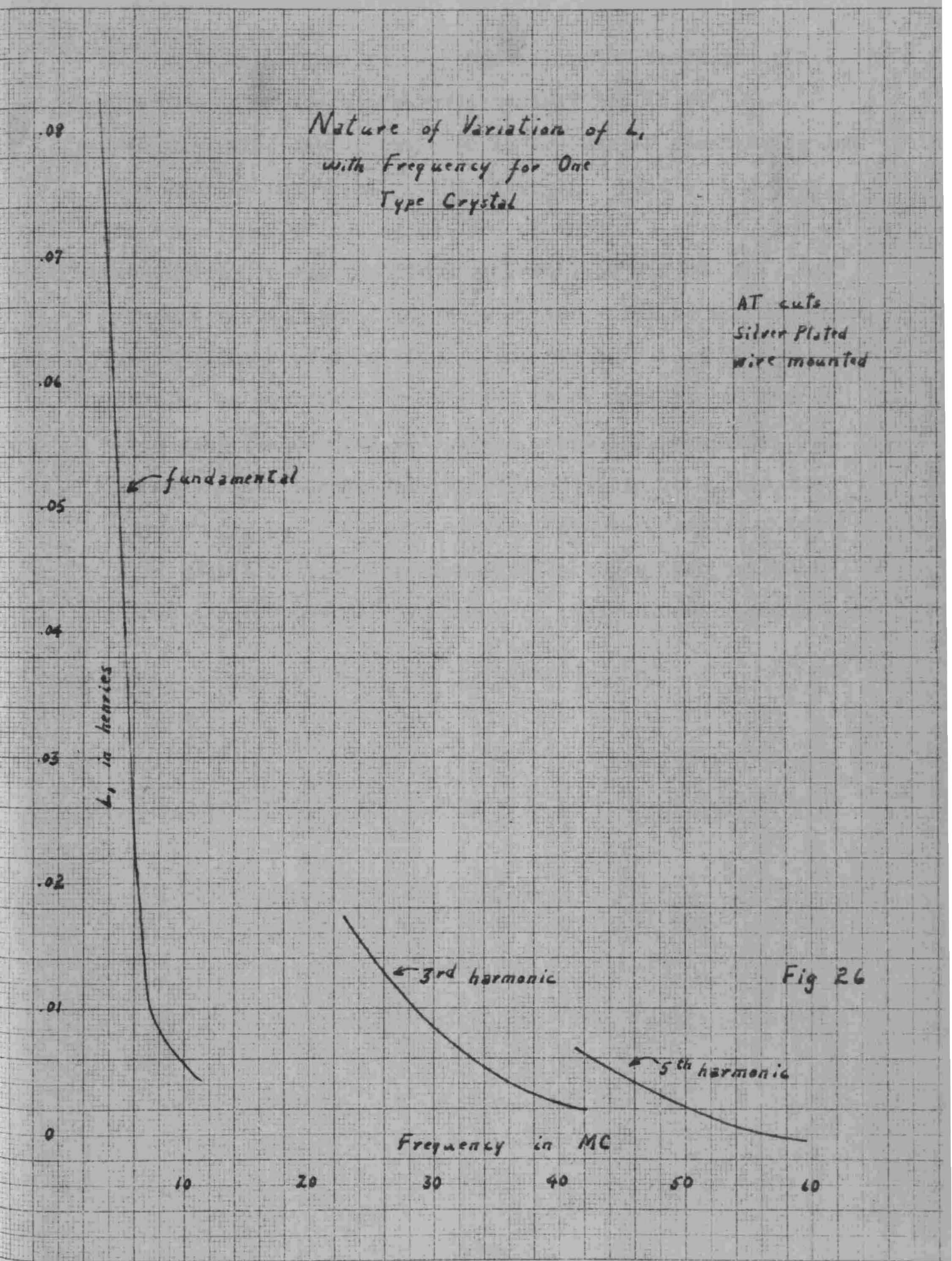
Freq.	R_1	Freq.	R_1
15 MC	20.5 ohms	30 MC	31.5 ohms
15 MC	19.6 ohms	35 MC	39 ohms
15 MC	51 ohms	35 MC	396 ohms
18.8 MC	26.5 ohms	40 MC	53 ohms
30 MC	43 ohms	45 MC	137 ohms
30 MC	30.6 ohms	45 MC	33.6 ohms
		45 MC	44.5 ohms

As high frequencies are considered, above 15MC, harmonic crystals are encountered. They will be third or fifth harmonics up to about 60M.C.

Results of measurements of an RCA type VC-1-E third harmonic crystal of nominal operating frequency 25.643 MC were as follows: At Fundamental frequency (8.3 MC)

C_L	f_s	$f_a - fs$	C_o	L_1	R_s
24 uuf	8.539096	4851 cycles	8uuf	.0101 h	13 ohms
At harmonic frequency (25.643 MC)					
17 uuf	25.642672	3117 cycles	8uuf	.0162 h	20 ohms

This crystal was of the type used in collecting the data of Fig. 26. From the data above, and Fig. 26 it will be seen that the L_1 value of the crystal considered at its fundamental frequency falls on the fundamental curve and the L_1 value obtained when measurements were made at the third harmonic frequency, falls on the third harmonic curve. It has been recognized that harmonic crystals show a smaller f_a-fs, an indication of increased stability with a change of load, than do fundamental crystals of the same operating frequency. Measurements show that harmonic crystals have unusually high L_1 values and corresponding lower C_1 values. This is in keeping with the smaller f_a-fs value obtained. This idea is presented in semi-quantitative form in Fig. 26.


Nature of Variation of L_1
with Frequency for One
Type Crystal
AT cuts
Silver Plated
wire mounted
fundamental
3rd harmonic
5th harmonic
Fig 26
L_1 in henries
.08
.07
.06
.05
.04
.03
.02
.01
0
Frequency in MC
10
20
30
40
50
60

If crystals are to be procured on the basis of specification of equivalent circuit parameters, it would seem that much will have to be done in the way of standardization on a limited number of optimum designs. This will decrease the variations of the parameters of crystals of the same frequency. It must be remembered that of all the 80-250KC crystals listed above, only one entirely failed to oscillate. The difficulty of establishing acceptable values for R_1, for example, is apparent. Only after those features of design which give the desired or optimum stability, activity, and dependability requirements have been established, and incorporated into a limited number of standard designs, will it be possible to set up meaningful general acceptance requirements for crystals in terms of their circuit parameters.

VI DISCUSSION OF MEASUREMENTS METHODS AND THE EQUIVALENT CIRCUIT

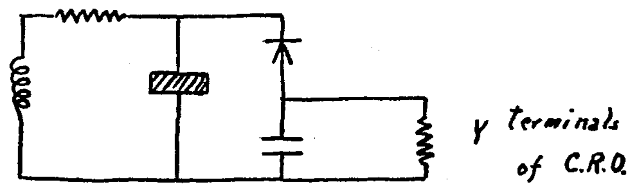
The circuits used in measuring the low frequency and medium frequency crystals, shown in Figs. 8 and 10 of Chapter II give results that are accurate enough for most laboratory purposes. Those used in measuring the high frequencies are subject to considerable error. If better accuracy is desired, it is apparent that some high frequency impedance measuring equipment, such as a slotted line, would be more desirable. The National Bureau of Standards uses a GR-516C R.F. Bridge to determine series resonance and R_1. Antiresonance is determined by a "Q" meter. Complementing this equipment they have a signal generator of unusual stability, much of whose output is derived from standard harmonics. A good r.f. bridge allows for reactances to be balanced out and frequencies of unity power factor located. At very high frequencies, the National Bureau of Standards has used slotted line technique. A brief description of this technique, from notes of J. S. Stoddard is given below.

The stable frequency generator already mentioned is used. Its frequency range is 1.25 to 320 MC, continously adjustable, with a power output of about .1 watt across 50 ohms. Stability is essential because these measurements require enough time to allow considerable frequency drift of an ordinary generator.

The r.f. power is transmitted to the crystal by a Rollins Company slotted line, and r.f. probe being used to measure relative voltage along the line. The voltage probed is amplified, detected and indicated by a well shielded Stoddart field intensity receiver and a meter. Also there is provision for attenuation of the probed voltage by means of a calibrated variable r.f. attenuator before the voltage reaches the receiver.

The crystal unit is fastened in a special holder to insure good contact of crystal terminals and good shielding at the load end of the line. Measurements are made with a low level of voltage across the crystal, to prevent variations due to IR heating effects in the crystal.

An approximate value of the frequency to be measured is first determined: The crystal is placed in a silicon-crystal diode detector circuit.



The circuit is coupled to a variable frequency oscillator and approximate resonance frequency detected by observation of transients on the oscilloscope. This allows the stable signal generator to be set up near the frequency to be measured.

The probe is placed about one half wave length from the crystal unit. Frequency is varied until a sharp voltage dip is observed. To be sure of getting the major dip and for accuracy, the crystal is replaced by a short and max. and min. points located accurately from the load end.

The crystal is replaced and the probe placed exactly half a wave length from the load. If necessary, the frequency is varied just enough to obtain a dip, indicating a frequency of maximum admittance of the crystal, or its R_1 L1 Cl C_o circuit.

The probe is then moved toward the load end until a minimum is found. The meter reading from the detector is recorded. The probe is moved toward the generator end one quarter of a wave length, to a maximum; attenuation is applied until the meter reads the same as it read at the minimum. The change in db is changed to voltage ratios, giving a Standing Wave Ratio. Knowing the change in minimum position with the crystal unit in and the maximum position with a short, allows calculation of the resistance and reactance components of the impedance of the crystal at the frequency of maximum admittance. The resistance component is R_1; the reactance component is the reactance of C_o.

Necessary equations are:

$$R_s = Z_o \frac{r}{(r \cos BL_{min})^2 + (\sin Bl_{min})^2}$$

$$X_s = jZo \frac{[1-r^2] \cos Bl \min \quad \sin Bl \min}{(r \cos Bl \min)^2 + (\sin Bl \min)^2}$$

$$B = \frac{2}{\lambda} \qquad R_1 = R_p = \frac{Z_s^2}{R_s} \qquad X_{Co} = X_p = \frac{Z_s^2}{X_s}$$

$\lambda_{min.}$ is distance from reference point to first min. toward generator end. Ref. Point is min. with shorted line.

r = standing wave ratio. Zo characteristic resistance of line.

The methods of high frequency crystal measurements described in Chapter II were used in this instance for three reasons:

(1) Although the scope of this study was such as to prohibit the refinement of a high frequency CI meter, it is believed that an instrument for higher than 12.5 MC (which is maximum for CI meter of Fig. 10) could be built which would give useful data on high frequency crystals.

(2) The very inadequacies of the equipment used in this study were in some cases quite instructive as to the nature of the equivalent circuit of the crystal. Part of the remainder of this chapter is devoted to a discussion of some of the problems that arose and how the inconsistencies were interpreted.

(3) No matter what equipment is used, a certain amount of confusion is likely to result if the terms series resonance and antiresonance are not defined and clarified for each set of measurement data. This point is also discussed in the following pages.

In one instance, a crystal of 60 MC nominal frequency was measured with the following results:

C_o	C_L	f_a-fs	L_1 calculated
17.7 uuf	92.1uuf	1300	.00147
	51	1800	.00170
	39	2000	.00206

Ideally, L_1 should be calculated as the same value, regardless of the C_L used in the measurement. The question arises as to which is more nearly correct, the small L_1 values or the large.

Suppose it is assumed that the larger values are tending toward the correct value of L_1.

Assume L_1 = .00206 henries.

Then C_1 = .0034uuf.

From (E_2) $\Delta f = \frac{f_s \quad C_1}{2} \cdot \frac{1}{(C_L + C_o)}$

If L_1 and C_1 are the assumed values, (E_2) predicts that Δf will be 1800 cycles, when C_L = 39 uuf.

When C_L = 51uuf, a smaller Δf is predicted, 1490 cycles.

When C_L = 92.1uuf, Δf predicted, 930 cycles.

These predicted values with the values actually measured are presented in tabular form:

C_L	Predicted Δf	Actual Δf	Error from Prediction in Per cent
39 uuf	1800 cycles	2000	11%
51	1490	1800	21%
92.1	930	1300	40%

This would indicate that since

$$L_1 = \frac{1}{4\pi^2 \, 2f \, \Delta f \, C_t}$$

and based on the guess that L_1 is nearer .00206 than to .00147, all values of Δf have been measured too high, say because f_a has been located at too <u>high</u> a frequency.

If on the other hand, L_1 is assumed to be about .00147 henries and C_1 = .0048 uuf, the following predicted and actual values result:

C_L	Predicted Δf	Actual Δf	Error from Prediction in Per cent
39 uuf	2540	2000	21%
51	2100	1800	14%
92.1	1310	1300	1%

This indicates that Δf has always been measured too small, i.e., f_a has been located at too <u>low</u> a frequency. However from observation of more reliable measurement data at lower frequencies, it seems that the Δf measured should have been much less than it was, favoring slightly the guess that f_a was located at too high a frequency.

This frequency was shown (Chapter II) to be the same as antiresonance if C_1 were in parallel with the crystal.

However, while the formula for L_1 was developed by neglecting R_1, what is really located for f_a, is a point of <u>minimum impedance</u>. Whether L_1 is .00147 henries or .00206 henries or neither, its approximate value is known, close enough that it is possible to investigate the question:

Where does the frequency of minimum impedance across the crystal in series with C_L fall with respect to the frequency given by

$$f_a = \frac{1}{2\pi} \sqrt{\frac{1}{L_1 C_1} + \frac{1}{L_1 C_t}} \quad ?$$

Impedances or admittances of several circuits representing different crystals and their C_L 's have been calculated and plotted, revealing the following:

(1). The actual f_a located, (the frequency of minimum impedance if C_L is in series with the crystal; maximum impedance if C_L is in parallel with the crystal) is <u>higher</u> than the f_a calculated by neglecting R_1; i.e. $\underline{\Delta}$ f is measured to great. Because $\underline{\Delta}$ f is always naturally greater for a small value of C_L, the percentage of error in L_1 which is incurred, is smaller for small values of C_L. See Fig. 27. Therefore, the <u>larger</u> values of L_1 are more nearly correct than the smaller.

(2). In the case of the crystal with circuit values taken as

L_1 = .0015 henries		(measured)
C_1 = .005 uuf		(measured)
R_1 = 10 ohms		(assumed)
C_o = 15uuf		(measured)
C_L = 40,65, 100uuf		(measured)

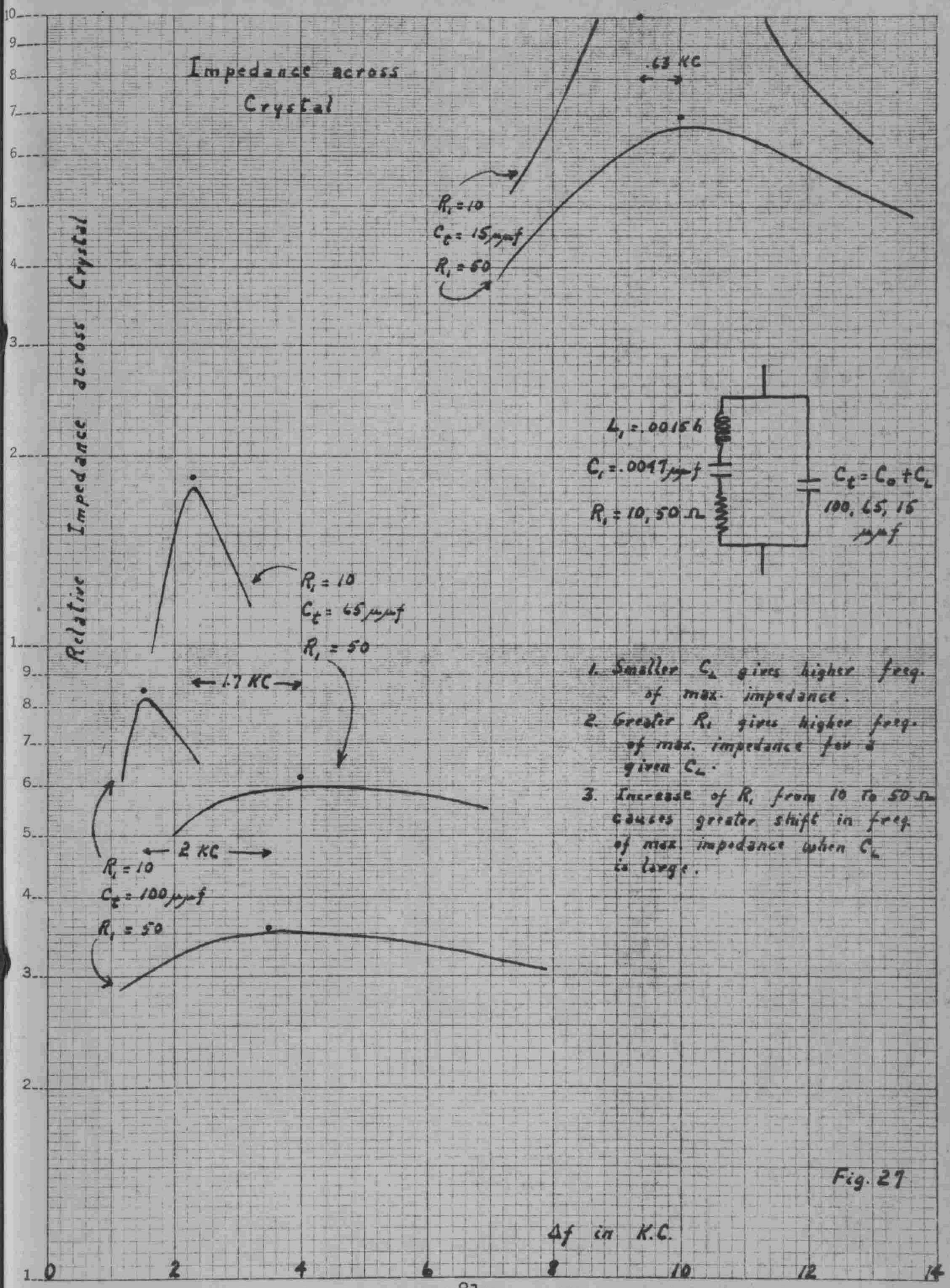
it is probable that if the value of L_1 is approximately correct, then R_1 is considerably more than 10 ohms; 30 or 40 ohms seems a more likely figure as this would cause discrepancies in L_1 more on the order of that actually encountered. See Fig. 28.

(3). Susceptance curves for a crystal, see Fig. 30, using different values of R_1 show that increasing R_1, decreases the magnitude of the negative susceptance of the crystal itself with no C_L. Since the negative susceptance of the crystal is increasing with frequency, and increasing R_1 raises this curve toward the zero axis, and since the positive susceptance of C_L is relatively constant at a certain level in the vicinity of resonance and antiresonance, increasing R_1 moves the point at which the negative susceptance of the crystal cancels the positive susceptance of C_L to a lower frequency. In fact if R_1 becomes large enough, the negative susceptance of the crystal will be decreased to that it can no longer cancel the positive susceptance of C_L. This is contained in the expression

$$(K_1) \qquad C_{t_{max}} = \frac{1}{4\pi f R_1}$$

As shown graphically in Fig. 31, but also can be calculated, if C_t = 100uuf, for the crystal shown in that figure, R_1 cannot be greater than about 13.2 ohms if the crystal is to be able to operate at an antiresonant frequency.

Saying that R_1 increasing, moves the frequency at which the negative susceptance of the crystal cancels the positive susceptance of the C_L to a lower frequency, seems to contradict the belief that f_a was measured too high because of the effects of R_1. It is sufficient here, to point out that Fig. 29 shows that the frequency of minimum admittance across the crystal and its C_L is higher than either f_1 or f_2.



Impedance across Crystal
Relative Impedance across Crystal
.63 KC
$R_1 = 10$
$C_t = 15\ \mu\mu f$
$R_1 = 50$
$L_1 = .0015\ h$
$C_1 = .0047\ \mu\mu f$
$R_1 = 10, 50\ \Omega$
$C_t = C_o + C_L$
100, 65, 15 $\mu\mu f$
$R_1 = 10$
$C_t = 65\ \mu\mu f$
$R_1 = 50$
1.7 KC
2 KC
$R_1 = 10$
$C_t = 100\ \mu\mu f$
$R_1 = 50$
1. Smaller C_L gives higher freq. of max. impedance.
2. Greater R_1 gives higher freq. of max. impedance for a given C_L.
3. Increase of R_1 from 10 to 50 Ω causes greater shift in freq. of max. impedance when C_L is large.
Δf in K.C.
0
2
4
6
8
10
12
14
1.
2.
3.
4.
5.
6.
7.
8.
9.
10.

Fig. 27

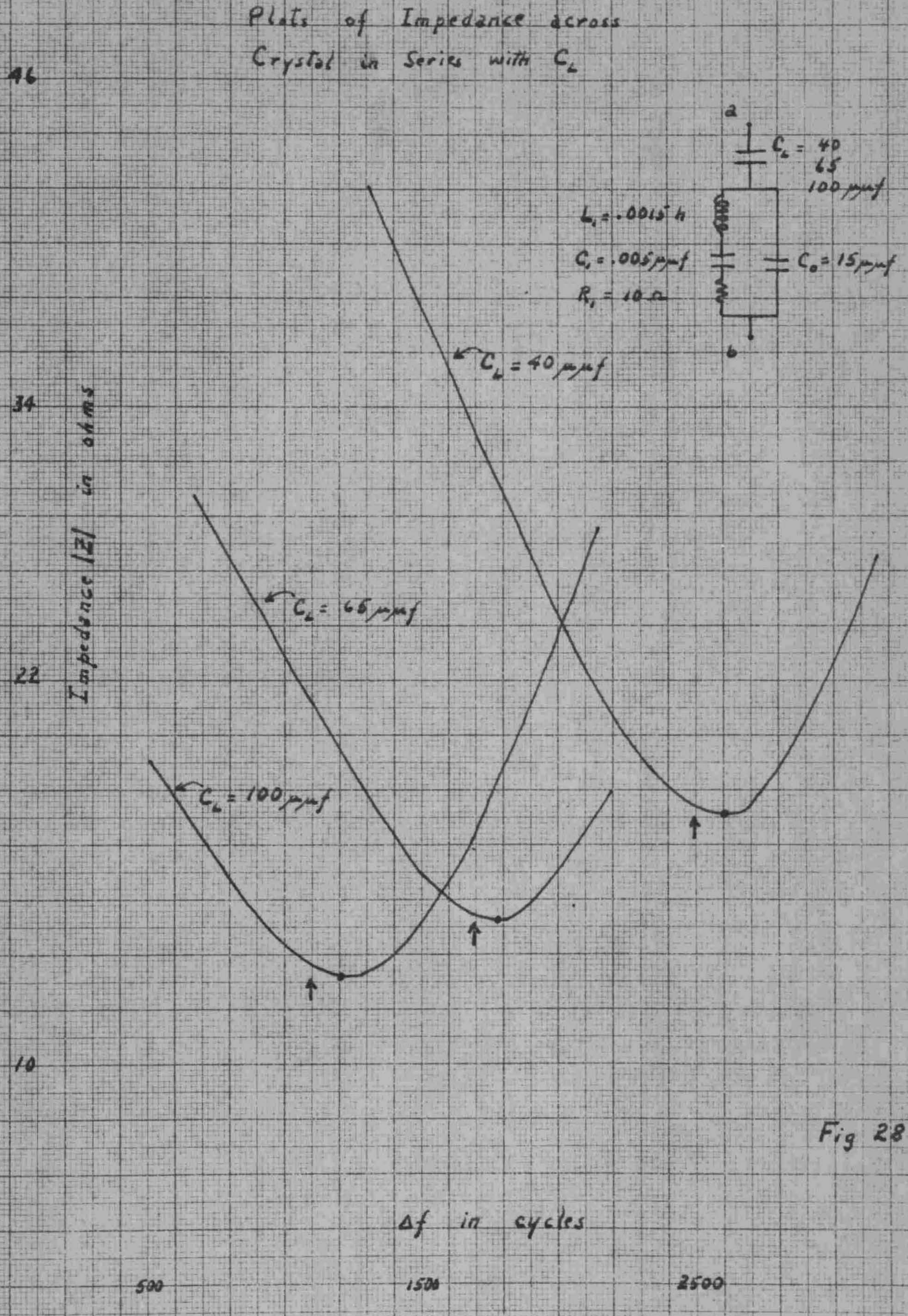

Plots of Impedance across
Crystal in Series with C_L
a
C_L = 40
65
100 μμf
L_1 = .0015 h
C_1 = .005 μμf
R_1 = 10 Ω
C_0 = 15 μμf
b
C_L = 40 μμf
C_L = 65 μμf
C_L = 100 μμf
46
34
22
10
Impedance |Z| in ohms
Δf in cycles
500
1500
2500

Fig 28

Admittance Curves of Crystal

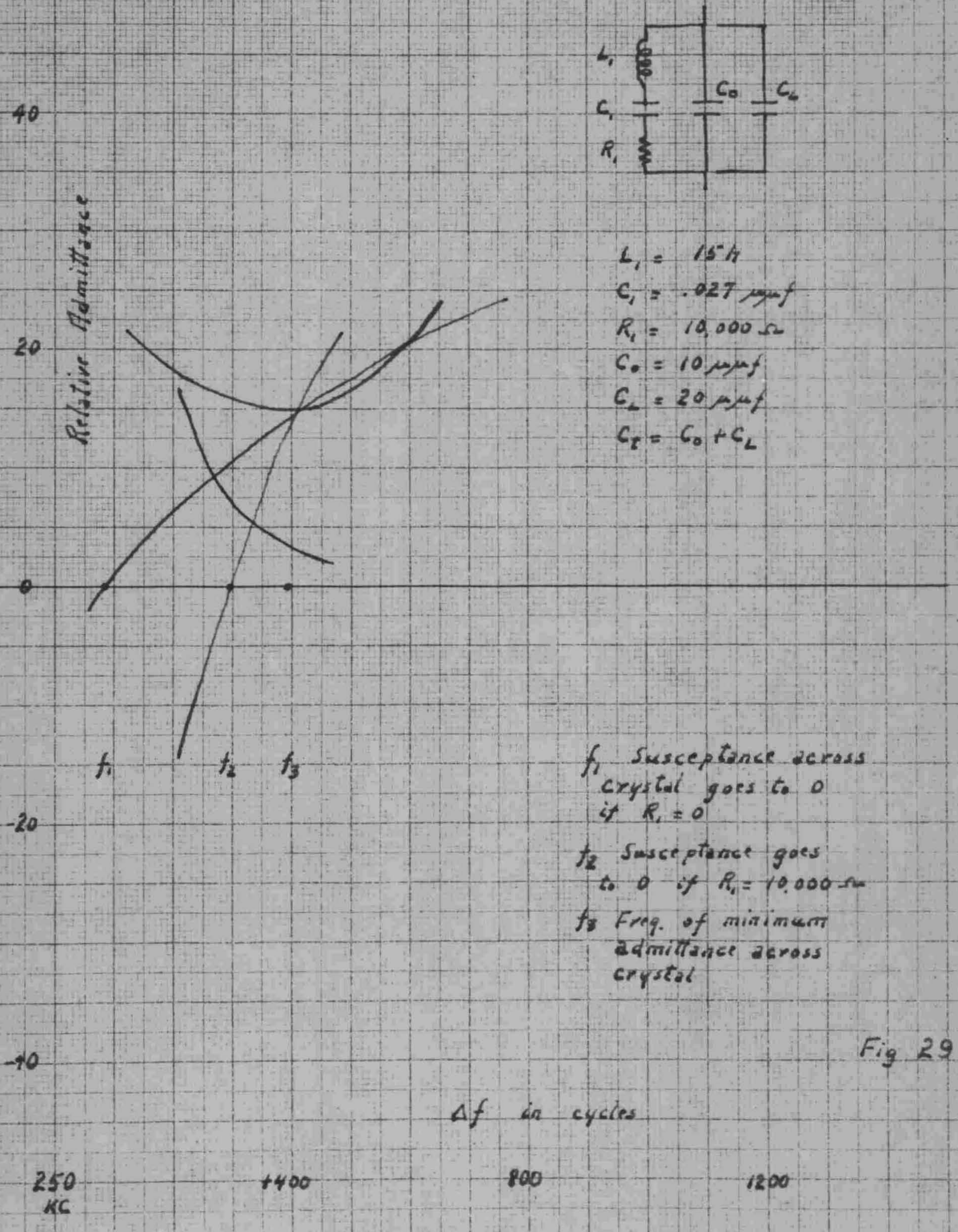


Fig 29

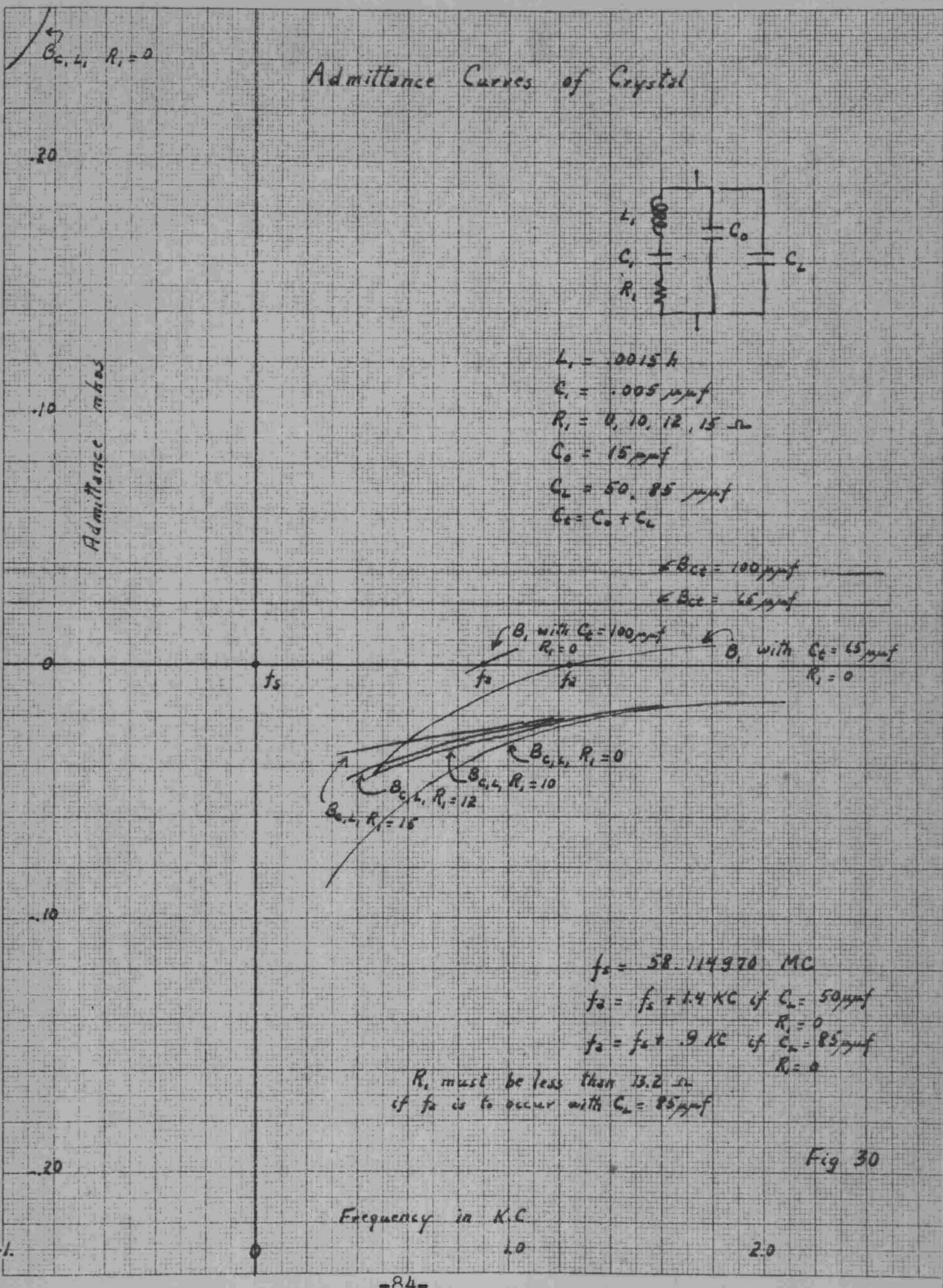
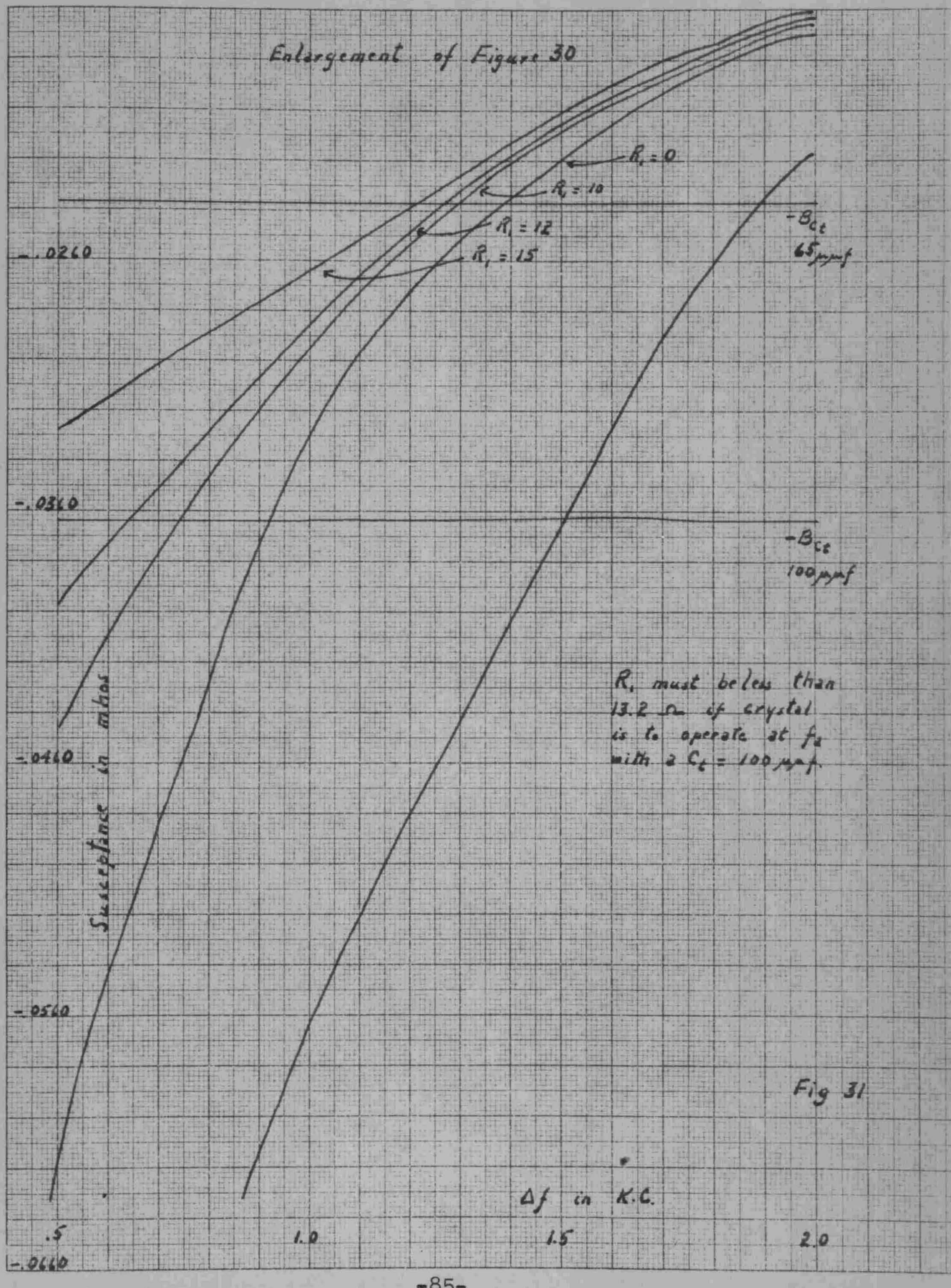
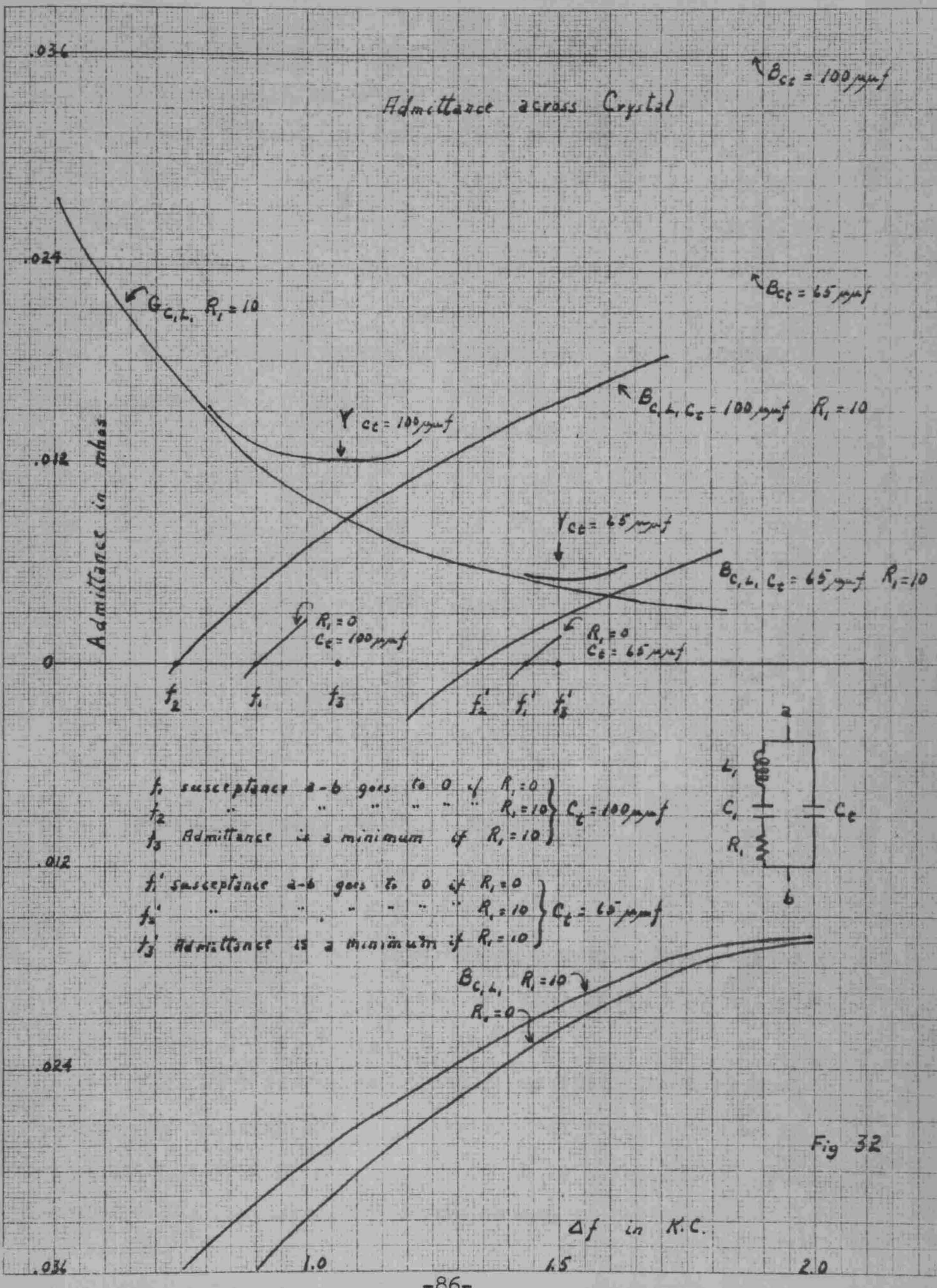

Admittance Curves of Crystal
$B_{C_1 L_1}$ $R_1 = 0$
L_1
C_1
R_1
C_0
C_L
$L_1 = .0015$ h
$C_1 = .005$ μμf
$R_1 = 0, 10, 12, 15$ Ω
$C_0 = 15$ μμf
$C_L = 50, 85$ μμf
$C_t = C_0 + C_L$
$B_{Ct} = 100$ μμf
$B_{Ct} = 65$ μμf
B_1 with $C_t = 100$ μμf $R_1 = 0$
B_1 with $C_t = 65$ μμf $R_1 = 0$
f_s
f_a
f_a
$B_{C_1 L_1}$, $R_1 = 0$
$B_{C_1 L_1}$, $R_1 = 10$
$B_{C_1 L_1}$, $R_1 = 12$
$B_{C_1 L_1}$, $R_1 = 15$
$f_s = 58.114970$ MC
$f_a = f_s + 1.4$ KC if $C_L = 50$ μμf $R_1 = 0$
$f_a = f_s + .9$ KC if $C_L = 85$ μμf $R_1 = 0$
R_1 must be less than 13.2 Ω
if f_a is to occur with $C_L = 85$ μμf
Admittance mhos
.20
.10
0
-.10
-.20
Frequency in K.C
-1.
0
1.0
2.0

Fig 30



Enlargement of Figure 30
$R_1 = 0$
$R_1 = 10$
$R_1 = 12$
$R_1 = 15$
$-B_{c_t}$ 65 $\mu\mu f$
$-B_{c_t}$ 100 $\mu\mu f$
-.0260
-.0360
-.0460
-.0560
-.0660
Susceptance in mhos
R_1 must be less than 13.2 Ω if crystal is to operate at f_2 with a $C_t = 100\ \mu\mu f$.
Δf in K.C.
.5
1.0
1.5
2.0

Fig 31

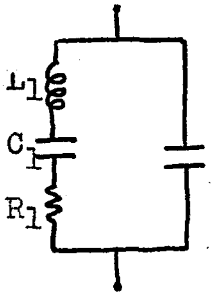


Admittance across Crystal
Admittance in mhos
.036
.024
.012
0
.012
.024
.036
$B_{C_t} = 100\ \mu\mu f$
$B_{C_t} = 65\ \mu\mu f$
$G_{C_1 L_1}\ R_1 = 10$
$B_{C_1 L_1 C_t = 100\ \mu\mu f}\ R_1 = 10$
$Y_{C_t = 100\ \mu\mu f}$
$Y_{C_t = 65\ \mu\mu f}$
$B_{C_1 L_1 C_t = 65\ \mu\mu f}\ R_1 = 10$
$R_1 = 0$
$C_t = 100\ \mu\mu f$
$R_1 = 0$
$C_t = 65\ \mu\mu f$
f_2 f_1 f_3 f_2' f_1' f_3'
f_1 susceptance a-b goes to 0 if $R_1 = 0$
f_2 " " " " " " $R_1 = 10$
f_3 Admittance is a minimum if $R_1 = 10$
$C_t = 100\ \mu\mu f$
f_1' susceptance a-b goes to 0 if $R_1 = 0$
f_2' " " " " " " $R_1 = 10$
f_3' Admittance is a minimum if $R_1 = 10$
$C_t = 65\ \mu\mu f$
a
b
L_1
C_1
R_1
C_t
$B_{C_1 L_1}\ R_1 = 10$
$R_1 = 0$
Δf in K.C.
1.0
1.5
2.0
Fig 32

Although most of the discussion has concerned the frequency of antiresonance, the frequency of series resonance is subject to some confusion also.

Suppose series resonance is defined as that frequency at which the reactive currents through the L_1 C_1 R_1 branch and the C_o branch are equal and opposite, giving unity power factor.

Consideration of the circuit reveals the following:



C_o

$$Y_o = j\omega C_o$$

$$Y_1 = R_1 - j X_1$$

$$Y_t = \frac{R_1}{R_1^2 + X_1^2} + \frac{j(R_1^2 + X_1^2 - X_{Co}X_1)}{X_{Co}R_1^2 + X_{Co}X_1^2}$$

For unity power factor, $X_1^2 - X_{Co}X_1 + R_1^2 = 0$

$$X_1 = X_{Co} \pm \sqrt{X_{Co}^2 - 4R_1^2}$$

Now this is a very interesting equation because it has one root if $X_{Co}^2 = 4R_1^2$ and two real roots if $X_{Co}^2 > 4R_1^2$ and no real roots if $X_{Co}^2 < 4R_1^2$.

Fig. 33 illustrates this information. For a crystal whose R_1 is about 32 ohms it shows that if the crystal is paralleled by a C_L giving a C_t of up to about 22,500 uuf, two "series resonance" frequencies are to be found and an impedance bridge will balance for <u>two</u> values of Rp. At a critical value of C_t <u>one</u> frequency will be measured, and for larger values of C_t, unity power factor cannot be observed. Fig. 33 is the result of experimental data by National Bureau of Standards.

It is apparent that the following distinct frequencies may be listed:

f_1 - a "resonant" frequency at which the admittance of the L_1 C_1 R_1 branch is a maximum.

f_2 - a "resonant" frequency at which the admittance across the entire L_1 C_1 R_1 C_o is a maximum.

f_3 - a "resonant" frequency at which unity power factor is achieved, when the L_1 C_1 R_1 C_o circuit appears as a pure resistance.

f_4 - an antiresonance frequency given by the resonant frequency of L_1 C_1 R_1 C_o all in series.

f_5 - an antiresonant frequency at which unity power factor is again achieved. (A C_L will normally be in parallel with C_o)

f_6 - an antiresonant frequency at which the admittance across L_1 C_1 R_1 C_o C_L is a minimum.

RF Bridge Measurements GT Crystal

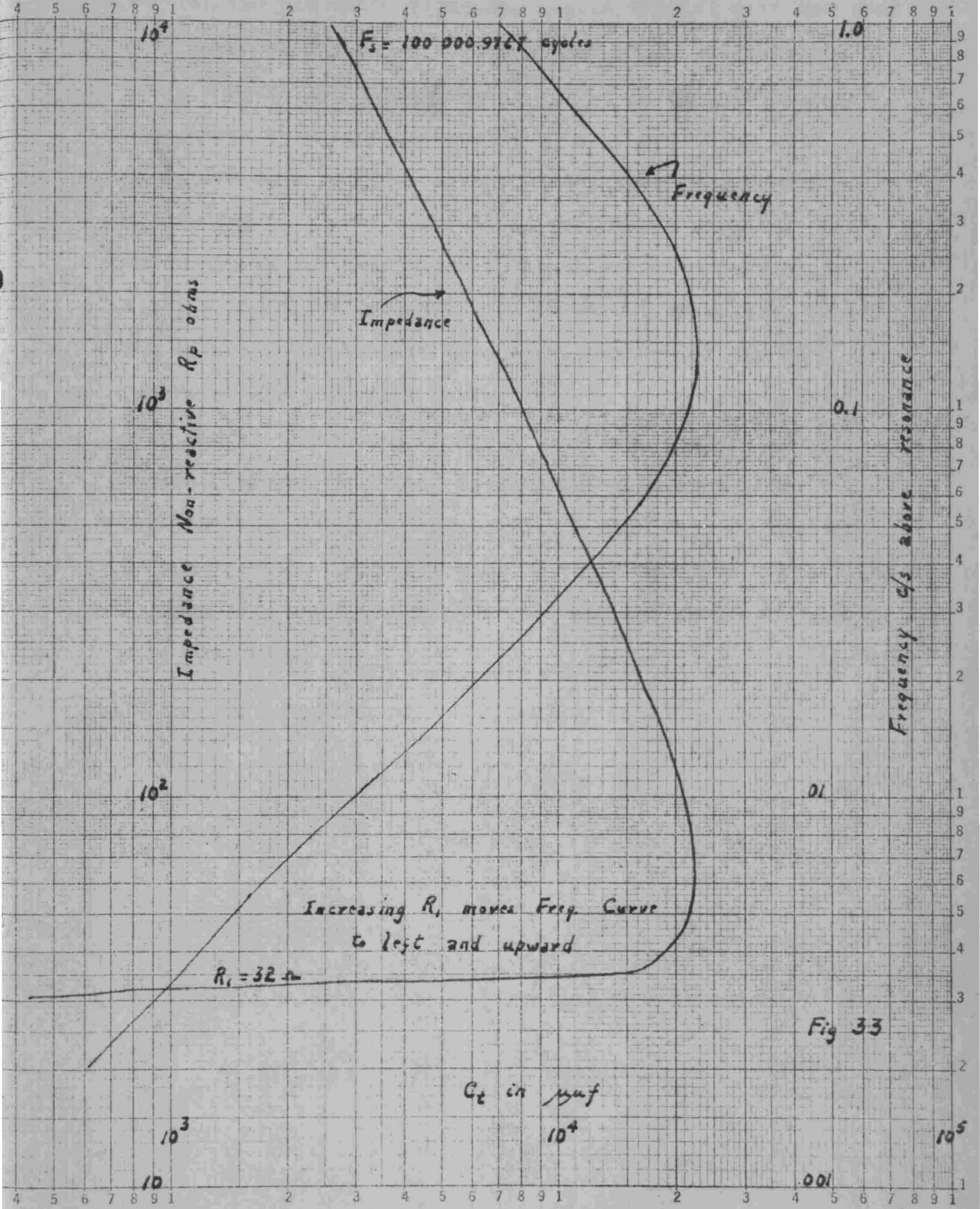


Fig 33

In the preceding pages the quartz crystal unit has been considered as a tank circuit composed of a series branch L_1 C_1 R_1 in parallel with a branch C_o. Relations among these parameters were examined and methods of measuring L_1 C_1 R_1 C_o f_s and f_a with known values of C_L, were presented. An experiment to show the effect of variations of C_o on stability and activity was described, pointing out that increasing C_o has the effect of increasing activity but decreasing stability. The values of equivalent circuit parameters of the same frequency and different design were shown to vary quite widely; even those of the same frequency and same design may vary a hundred percent or more. Measurement data at different frequencies was given to illustrate the magnitude of the results to be expected. Finally, some of the difficulties encountered in measuring and specifying series and antiresonant frequencies were discussed.

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APPENDIX

Frequency data, headed Plates No. 1, 2, 3, 4, and 5 was taken on the C.I. meter of Fig. 10 in the block setup of Fig. 6. Crystal current was maintained at 10 m. a. Data concerning voltage developed across these crystals was taken using circuits of Figs. 15 and 16. This data was used in the computations of the curves of Figs. 19, 21, 22, and 23.

Data headed Reeves Hoffman Crystals was taken on the CI meter of Fig. 10. All C_o values were measured on the Q-Meter. This data was used in Figs. 2 and 18.

The data headed A Frequency Series of Crystals was taken with the CI meter of Fig. 10 up through 8 MC. For higher frequencies, the circuits of Figs. 11, 12, 13, and 14 in block setup of Fig. 7 were used. The data was used in Fig. 26.

Plate No. 1 (140 mils diameter)

Xtal No.	C_o uuf	f_s MC	R_1 ohms	f_a-27 MC	f_a-32 MC	f_a-37 MC	f_a-f_s cps
2	-						
3	2.65	9.010059	17		9.011963		1904
4	3.2	8.994071	18		8.995777		1706
7	2.7	8.971157	17		8.973058		1901
8	2.7	8.994234	20		8.996209		1975
9	2.7	8.969882	19		8.971854		1972
	2.7		18	--Average Values--			1892

	L_1h	C_1 uuf	(f_a-32)-(f_a-27)	(f_a-37)-(f_a-32)	A_1 100	A_1 150	A_1 175	AVT 175
2	-							
3	.0123	.0147			34	48	73	
4	.0234	.0134			35	54	69	
7	.0214	.0146			35	56	73	
8	.0205	.0152			32	50	67	
9	.0206	.0152			36	50	68	
	.0214	.0146	--Average--		34	52	70	

The 34,52,70 correspond to 4.7,6.3,7.8 volts

f_a-27 denotes antiresonance when C_L is 27 uuf.

f_a-f_s denotes difference between antiresonance when C_L is 32 uuf and series resonance.

A-100 denotes meter reading of A_1 on Fig.15, plate voltage 100. A_1 was calibrated and average values are converted into rms volts across Xtal.

AVT-175 gives rms volts developed across Xtal in circuit of Fig. 16.

Plate No. 2 (173 mils diameter)

Xtal No.	C_o uuf	f_s MC	R_1 ohms	f_a-27 MC	f_a-32 MC	f_a-37 MC	f_a-f_s cps
2	4.0	8.989014	14	8.991512	8.991213	8.990935	2209
3	-	9.005752	-	9.008313	9.008007	9.007728	2255
4	3.8	8.992967	15	8.995447	8.995152	8.994875	2185
5	4.1	8.994255	12	-	8.996576	-	2321
6	4.2	8.972430	14	-	8.989533	-	2294
7	4.2	8.970427	14	8.973102	8.972774	8.972474	2347
8	4.3	8.992164	12	8.994815	8.994480	8.994156	2316
9	4.0	8.969861	14	8.972263	8.971978	8.971694	2117
	4.1		13.6	--Average Values--			2259

	L_1h	C_1 uuf	$(f_a-32)-(f_a-27)$	$(f_a-37)-(f_a-32)$	A_1 100	A_1 150	A_1 175	AVT 175
2	.0177	.0177	299	278	46	72	86	
3	.0172	.0181	306	279	-	-	-	
4	.0172	.0174	295	277	45	75	84	
5	.0168	.0186	-	-	-	-	-	
6	.0170	.0185	-	-	-	-	-	
7	.0166	.0189	328	300	49	70	80	
8	.0167	.0187	335	324	52	76	93	
9	.0185	.0170	285	274	47	68	80	
	.0172	.0181	*	--Average--	48	73	83	

The 48,73,83 correspond to 6,8.1,8.9 volts

*Average change in f_a for change in C_L of 5 uuf is 298 cps.

Average change in f_a for change in C_L of 1 uuf is 60 cps.

Average change in f_a for change in C_L of 1 uuf is .00067%

See data sheet on Plate No. 1 for explanation of headings.

Plate No. 3 (212 mils diameter)

Xtal No.	C_o uuf	f_s MC	R_1 ohms	f_a-27 MC	f_a -32 MC	f_a-37 MC	f_a-f_s cps
1	5.6	8.988730	7	8.992143	8.991746	8.991363	3016
2	5.8	8.972834	7	8.976390	8.975964	8.975550	3130
3	5.5	9.001362	10	9.004860	9.004443	9.004056	3081
4	5.4	8.978533	9	8.982156	8.981746	8.981326	3213
5	5.5	8.977930	9	8.981338	8.980933	8.980587	3003
6	5.6	8.955327	9	8.958858	8.958470	8.958083	3143
7	5.5	8.952477	9	8.955905	8.955510	8.955151	3033
8	5.4	8.979715	9	8.983315	8.982899	8.982520	3184
9	5.7	8.951257	8	8.954656	8.954263	8.953870	3006
	5.6		8.6	--Average Values--			3090

	L_1h	C_1 uuf	(f_a-32)-(f_a-27)	(f_a-37)-(f_a-32)	A_1 100	A_1 150	A_1 175	AVT 175
1	.0124	.0252	397	383	64	100	112	
2	.0119	.0264	426	414	61	96	110	
3	.0121	.0257	417	387	50	83	96	
4	.0117	.0267	410	420	54	85	100	
5	.0125	.0251	405	346	54	85	99	
6	.0119	.0264	388	387	55	90	104	
7	.0124	.0254	395	359	57.5	91	106	
8	.0118	.0266	416	379	62	99	115	
9	.0125	.0253	393	407	58	89	105	
	.0121	.0259	* --Average--		57	91	105	22.4volts

The 57,91,105 correspond to 6.8,9.6,10.8 volts

*Average change in f_a for change in C_L of 5 uuf is 402cps.

Average change in f_a for change in C_L of 1 uuf is 80 cps.

Average change in f_a for change in C_L of 1 uuf is .00089%

See data sheet on Plate No. 1 for explanation of headings.

Plate No. 4 (250 mils diameter)

Xtal No.	C_o uuf	f_s MC	R_1 ohms	f_a-27 MC	f_a-32 MC	f_a-37 MC	f_a-f_s cps
2	7.6	8.966964	7	8.971544	8.971046	8.970547	4082
3	7.6	8.987833	8	8.992345	8.991874	8.991380	4041
4	7.6	8.966347	7	8.971115	8.970620	8.970100	4273
5	7.6	8.963630	7	-	8.967750	-	4120
6	7.6	8.941417	7	-	8.945512	-	4095
7	-	8.938272	7	-	8.942372	-	4100
8	7.9	8.960893	8	8.965625	8.965100	8.964605	4207
9	7.4	8.940622	7	8.945222	8.944713	8.944231	4091
	7.6		7.2	--Average Values--			4126

	L_1h	C_1 uuf	(f_a-32)-(f_a-27)	(f_a-37)-(f_a-32)	A_1 100	A_1 150	A_1 175	AVT 175
2	.00873	.0361	498	499	65	100	127	
3	.00880	.0356	471	494	70	109	124	
4	.00832	.0377	495	520	67	105	115	
5	.00864	.0364	-	-	-	-	-	
6	.00870	.0362	-	-	-	-	-	
7	.00871	.0363	-	-	-	-	-	
8	.00861	.0373	525	505	60	105	114	
9	.00879	.0361	509	482	65	104	124	
	.00866	.0365	* --Average--		65	104	122	30 volts

The 65,104,122 correspond to 7.4,10.7,12.2 volts

*Average change in f_a for change in C_L of 5 uuf is 500cps.

Average change in f_a for change in C_L of 1 uuf is 100cps.

Average change in f_a for change in C_L of 1 uuf is .0011%.

See data sheet on Plate No. 1 for explanation of headings.

Plate No. 5 (312 mils diameter)

Xtal No.	C_o uuf	f_s MC	R_1 ohms	f_a -27 MC	f_a -32 MC	f_a -37 MC	f_a-f_s cps
2	12	8.949833	6	8.956018	8.955390	8.954820	5557
3	11.7	8.966679	6	8.972855	8.972282	8.971674	5603
4	11.7	8.956470	6	8.962662	8.962077	8.961467	5607
5	11.5	8.962630	6	-	8.968222	-	5592
6	11.8	8.941027	6	-	8.946644	-	5617
7	11.7	8.938237	5.5	8.944462	8.943854	8.943250	5622
8	11.7	8.960756	6	-	-	-	-
9	11.4	8.940194	7	8.946420	8.945798	8.945224	5604
	11.7		6 --Average Values--				5602

	L_1h	C_1 uuf	(f_a -32)-(f_a -27)	(f_a -37)-(f_a -32)	A_1 100	A_1 150	A_1 175	AVT 175
2	.00576	.0546	628	570	75	120	141	
3	.00572	.0548	573	608	75	115	134	
4	.00575	.0547	585	610	75	115	134	
5	.00577	.0545	-	-	-	-	-	
6	.00575	.0550	-	-	-	-	-	
7	.00575	.0550	603	609	75	122	143	
8	.00574	.0548	-	-	-	-	-	
9	.00582	.0544	622	574	73	120	140	
	.00576	.0547	* --Average--		74	120	140	36 volts

The 74,120,140 correspond to 8.1,12.1,13.8 volts.

*Average change in f_a for change in C_L of 5 uuf is 598cps.

Average change in f_a for change in C_L of 1 uuf is 120cps.

Average change in f_a for change in C_L of 1 uuf is .0013%.

See data sheet on Plate No. 1 for explanation of headings.

Reeves-Hoffman Crystals

Xtal No.	Freq. KC	R_1 ohms	f_s	C_L is 32uuf $f_a - f_s$	ΔC_L is 5uuf av. f_s	C_o uuf
1	3580	32	3578.827	1073	117 (cps)	5.5
2	3600	42	3598.772	1168	153	5.5
3	3600	37	3598.801	1149	210	5.5
4	3900	26	3898.600	1310	165	5.5
5	3900	25	3698.706	1294	171	5.8
6	4165	38	4162.231	1375	177	6.
7	4165	24	4163.480	1413	140	6.
8	4450	17	4448.098	1702	202	6.5
9	4450	22	4448.305	1495	250	6.2
10	4698.7	16	4697.008	1585	206	6.2
11	4730	18	4728.264	1736	210	6.1
12	4730	16	4728.138	1726	210	6.2
13	4761.3	18	4759.440	1760	231	6.1
14	4850	18	4847.965	1915	242	6.5
15	5150	9	5147.900	2005	256	6.5
16	5150	10	5147.705	2175	279	6.7
17	5615	17	5612.576	2376	301	7.
18	5615	23	5612.515	2408	289	7.
19	6050	12	6047.052	2818	340	7.3
20	6050	13	6047.155	2700	327	7.6
21	6451	8	6447.740	3220	374	7.9
22	6500	7	6496.727	3273	377	8.2
23	6500	9	6496.474	3251	391	8.
24	6549	7	6545.630	3242	480	7.9
25	6950	10	6946.265	3512	414	8.5
26	6950	6	6946.170	3655	430	9.
27	7400	14	7396.075	3925	461	9.
28	7400	16	7396.198	3802	444	8.8
29	7882.5	9	7878.110	4373	510	9.8
30	7882.5	8	7878.107	4243	499	9.5
31	8050	7	8046.223	3723	445	7.9
32	8050	11	8046.082	3768	450	7.7
33	8232.5	8	8227.639	4489	516	9.5
34	8232.5	8	8227.607	4369	502	9.5
35	8800	bad				
36	8800	bad				
37	9188.35	6	9182.885	5346	599	10.7
38	9265	5	9260.000	4670	542	9.

Xtal No.	Freq. kC	R_1 ohms	f_s	C_L is 32uuf $f_a - f_s$	ΔC_L is 5uuf av. f_s	C_0 uuf
39	9265	5	9269.193	5378	578	10.9
40	9341.65	5	9337.718	4928	553	9.1
41	9650	5	9644.310	5640	609	11.3
42	9650	5	9644.930	5070	575	9.5
43	9800	4	9794.750	5250	582	9.6
44	9800	5	9794.016	6014	636	11.4
45	10100	4	10093.457	6503	692	12.2
46	10100	5	10093.614	6386	685	11.8
47	10400	4	10394.375	5501	623	9.8
48	10400	4	10394.663	5522	617	9.8
49	10530	5	10524.062	5601	620	9.6

Reeves-Hoffman Crystals
Change in f_a by Changing C_L by 5uuf

Xtal No.	Increase C_L 5uuf	%Change in f_a	Decrease C_L 5uuf	%Change in f_a	C_o uuf	Nom. Freq.KC
31	421	.00524	470	.00585	7.9	8050
32	441	.00549	460	.00571	7.7	8050
38	534	.00576	550	.00594	9.0	9265
40	541	.00579	576	.00617	9.1	9341.65
42	570	.00590	580	.00601	9.5	9650.
43	574	.00585	591	.00604	9.6	9800.
47	611	.00590	634	.00610	9.8	10400
48	615	.00591	620	.00596	9.8	10400
49	613	.00581	628	.00595	9.6	10530
		(Low C_o Group)				
29	488	.00619	532	.00675	9.8	7882.5
30	480	.00609	514	.00651	9.5	7882.5
33	490	.00595	542	.00659	9.5	8232.5
34	476	.00579	528	.00641	9.5	8232.5
37	588	.00640	605	.00659	10.7	9188.35
39	560	.00604	597	.00644	10.9	9265.
44	619	.00631	654	.00666	11.4	9800.
45	679	.00672	706	.00700	12.2	10100
46	633	.00626	738	.00730	11.8	10100
		(High C_o Group)				

A Frequency Series of Crystals

Xtal	Freq.MC	L_1 h	C_o uuf	R_1 ohms	
a	3.5	.085	5	35	Fundamental
b	5.	.030	6	15	Fundamental
c	6.	.020	7	12	Fundamental
d	7.	.012	8	8	Fundamental
e	8.	.010	8	8	Fundamental
f	25.	.016	8	20	Third Harmonic
g	42.	.004	12	40	Third Harmonic
h	60.	.0015	17	40	Fifth Harmonic

Data used in Fig. 26.